

ELEC2400, Assignment Number 1

1. (20 marks Total)

(a) Sketch the following signals and express them in the form

$$x(t) = f_1(t)\mathbf{1}(t - t_1) + f_2(t)\mathbf{1}(t - t_2) + \dots$$

i. (5 marks)

$$x(t) = \begin{cases} 1 - |t| & ; |t| < 1 \\ 0 & ; \text{Otherwise} \end{cases}$$

ii. (5 marks)

$$x(t) = \begin{cases} \cos(t) & ; |t| < \pi \\ |t/\pi| - 1 & ; |t| \in [\pi, 2\pi] \\ 0 & ; \text{Otherwise} \end{cases}$$

iii. (5 marks)

$$x(t) = \begin{cases} \exp(3t) & ; |t| < 3 \\ t^2 - 9 & ; |t| \in [3, 6] \\ 0 & ; \text{Otherwise} \end{cases}$$

(b) (5 marks) Derive differential equation relating voltage $v_0(t)$ to voltage $v_i(t)$ in the circuit of figure 1. Assume that the op-amp is ideal (infinite differential gain and infinite input impedance).

2. (30 marks Total) Compute and sketch the convolution

$$y(t) = [h \otimes u](t) = \int_{-\infty}^{\infty} h(t - \sigma)u(\sigma) d\sigma$$

for the following cases

(a) (10 marks)

$$h(t) = \begin{cases} 1 - t & ; t \in [0, 2] \\ t - 3 & ; t \in (2, 3] \\ 0 & ; \text{Otherwise} \end{cases}$$

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; \text{Otherwise} \end{cases}$$

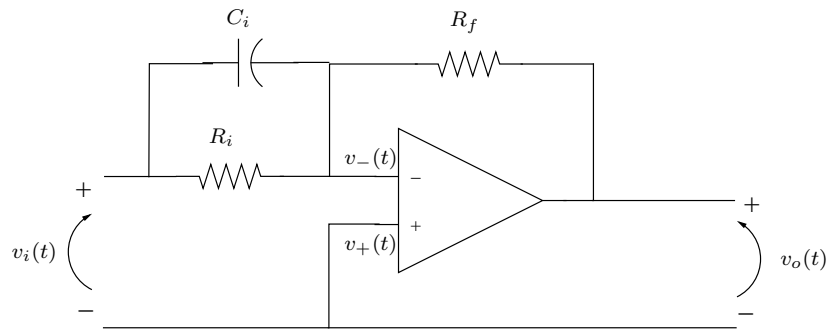


Figure 1: Circuit considered in question 1b

(b) (10 marks)

$$h(t) = \begin{cases} 1 & ; t \in [-1, 3] \\ 0 & ; \text{Otherwise} \end{cases}$$

$$u(t) = \begin{cases} 1 & ; t \in [0, 2] \cup [3, 4] \\ 0 & ; \text{Otherwise} \end{cases}$$

(c) (10 marks)

$$h(t) = \begin{cases} 1 & ; |t| \leq 1 \\ 0 & ; \text{Otherwise} \end{cases}$$

$$u(t) = \sum_{k=-\infty}^{\infty} \delta(t - Tk)$$

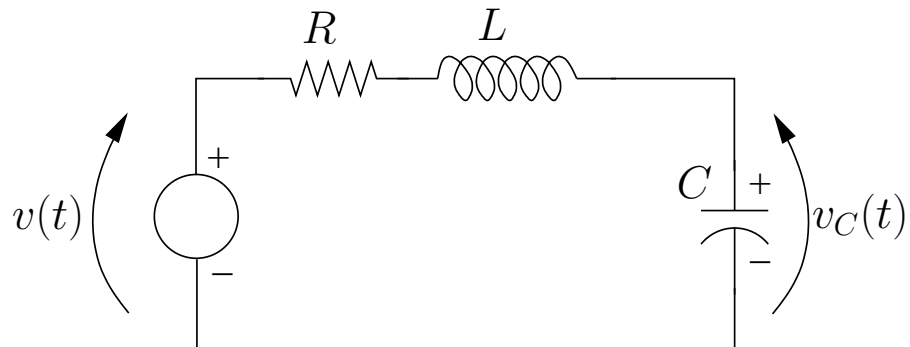
Here, δ is the Dirac delta function, and the question should be answered for the case $T = 4$ and $T = 1$.

3. (25 marks total) Suppose a linear system has an impulse response

$$h(t) = \begin{cases} 2e^{-t/2} \sin 3t & ; t \geq 0 \\ 0 & ; \text{Otherwise} \end{cases}$$

- (a) (6 marks) Sketch the impulse response.
- (b) (9 marks) Suppose this system is driven by an arbitrary input $u(t)$, write down an expression for what the output $y(t)$ of the system will be.
- (c) (10 marks) Is this a causal or a non-causal linear system? Explain your answer in terms a convolution relationship.

4. (25 marks total) Consider the circuit shown below



- (a) (5 marks) Derive a differential equation relationship between the voltage source signal $v(t)$ and the signal $v_R(t)$ which is the voltage across the resistor.
- (b) (5 marks) Present an observer canonical form state-space description of this same differential equation relationship.
- (c) For the particular case of

$$C = 25000\mu\text{F}, \quad L = 10\text{H}, \quad R = 50\Omega$$

provide an expression for the matrix exponential $e^{\mathbf{A}t}$ associated with the preceding state space realisation.

- (d) (5 marks) Use the results of the preceding two parts to derive the system impulse response $h(t)$.
- (e) (10 marks) Again using the results of preceding parts, use state-space methods to calculate and plot a the signal $v_R(t)$ that, at time $t = 0$ satisfies

$$v_R(0) = -2\text{V}, \quad \left. \frac{d}{dt}v_R(t) \right|_{t=0} = -1\text{V/s}$$

when $v(t)$ is the signal

$$v(t) = \begin{cases} 1 & ; |t - 1.5| \leq 0.5 \\ 0 & ; \text{Otherwise} \end{cases}$$