ELEC2400, Assignment Number 2

1. (**30 marks total**)

   (a) A common effect in communications, radar, and sonar is *multipath*. This occurs when there are multiple paths from a transmitter to a receiver, each with a different path length and propagation delay. The result is that the receiver gets multiple interfering copies of the message. We can model the simple case of two paths as a system in which the output signal $y(t)$ and the input signal $x(t)$ can be related through the following equation

   $$y(t) = x(t) + ax(t-T),$$

   where a is a constant weighting factor, and T is the difference between the propagation delays along the two paths.

      i. (**3 marks**) Compute the transfer function $H(s)$ relating the output $Y(s) = \mathcal{L}\{y(t)\}$ and the input $X(s) = \mathcal{L}\{x(t)\}$.

      ii. (**4 marks**) Compute and sketch the impulse response $h(t)$ relating $y(t)$ and $x(t)$.

   (b) A continuous-time signal $x(t)$ has Fourier transform $X(\omega)$. Express the Fourier transform of the following signals in terms of $X(\omega)$.

      i. (**3 marks**)
      $$y(t) = x(2t+5);$$

      ii. (**3 marks**)
      $$z(t) = x(t) \circledast x(t).$$

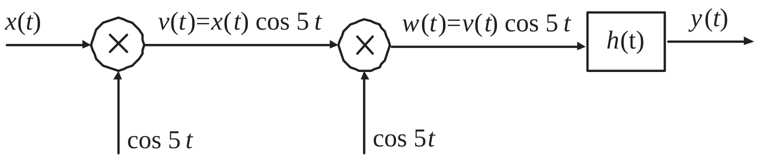

Figure 1: *System diagram considered in question 1c.*

   (c) (**17 marks**) Consider the system in Figure 1. The Fourier transforms of $h(t)$ is given by:

   $$H(\omega) = \begin{cases} 3 & ; 0 \leq |\omega| \leq 3 \\ 0 & ; \text{Otherwise} \end{cases}$$

   The Fourier transform of the input signal $x(t)$ takes the form

   $$X(\omega) = \begin{cases} 2 - |\omega| & ; |\omega| \leq 2 \\ 0 & ; \text{Otherwise} \end{cases}$$

   Sketch $V(\omega)$, $W(\omega)$ and $Y(\omega)$, which are the Fourier transforms of $v(t)$, $w(t)$ and $y(t)$, respectively.

2. (**45 marks total**) In the following, assume an ideal system.

   (a) (**10 marks**) With regard to the following signal,

   $$x(t) = 8\text{sinc}(36t)$$

   plot the ideally sampled signal and its amplitude spectrum for sampling frequencies of 24Hz and 45Hz. What is the minimum sampling rate that can be used so that the samples obtained can be used to reconstruct the signal?

   (b) (**15 marks**) With regard to the following signal,

   $$y(t) = \text{sinc}(5t)\text{sinc}\left(\frac{t}{2}\right)$$

   plot the ideally sampled signal and its amplitude spectrum for sampling frequencies of 4Hz, 5.5Hz and 10Hz. What is the minimum sampling rate that can be used so that the samples obtained can be used to reconstruct the signal?

   (c) Consider the following pulse signal

   $$x(t) = \begin{cases} 1 & ; t \in [0, 2] \\ 0 & ; \text{Otherwise} \end{cases}$$

   and suppose that samples of it are derived according to

   $$x_k = x(t)|_{t=k\Delta} \, .$$

   i. (**10 marks**) Denote by $x_R(t)$ a version of $x(t)$ that is reconstructed from the samples x_k. Provide a formula for $x_R(t)$ that arises from using a perfect band-limited reconstruction filter with bandwidth equal to half the sampling frequency.

   ii. (**10 marks**) Use MATLAB to plot the reconstructed signal for the two cases of $\Delta = 1$ and $\Delta = 1/4$ second.

3. (**25 marks total**)

   (a) The Transfer Function of a system is given as

   $$H(\omega) = \frac{10}{s^2 + 120}$$

   i. (**4 marks**) Compute the output $y(t)$ of this system when the input $x(t)$ is given as

   $$x(t) = 2 + 2\cos(10t + \pi/2).$$

   ii. (**4 marks**) Sketch the magnitude frequency response $|H(j\omega)|$.

   (b) An ideal linear-phase low-pass filter has frequency response function

   $$H(\omega) = \begin{cases} 6e^{-j2\omega} & ; |\omega| \leq 3 \\ 0 & ; |\omega| > 3. \end{cases}$$

   i. (**2 mark**) Plot the magnitude frequency response of this filter.

   ii. (**4 marks**) Compute the impulse response of this filter.

iii. (**6 marks**) Compute the output response $y(t)$ when the input $x(t)$ is given by

$$x(t) = \text{sinc}\left(\frac{2t}{\pi}\right)$$

for $-\infty < t < \infty$. (Recall that $y(t) = \text{sinc}(t)$ implies $Y(\omega) = 0.5\Pi_\pi(\omega)$ where $\Pi_\pi(\omega)$ is a rectangle of unit height over the range $\omega \in [-\pi, \pi]$).

(c) Consider the following system

$$y(t) = \begin{cases} x(t+1) & ; |x(t)| \leq 10 \\ 10 & ; |x(t)| > 10 \end{cases}$$

Determine the following properties of this system

i. (**1 mark**) Causal or non-causal?
ii. (**1 mark**) Linear or non-linear?
iii. (**1 mark**) Time Invariant or time varying?
iv. (**1 mark**) With memory or Memoryless?
v. (**1 mark**) Stable or unstable?