

# ELEC2400 Signals & Systems

## Chapter 2. Signal Types and Operations

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## 2. Signal Types and Operations

### Outline

- What is a signal?
- Types of Signals
- Operations on Signals
- Concluding Summary

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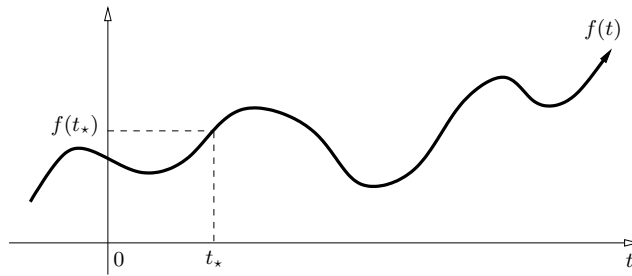
## What is a signal?

It is the time evolution of a quantity, for example:

- The price of a share in a publically listed company;
- The level of water in a reservoir;
- The speed of a car;
- The temperature in a room;
- The voltage driving the speaker in a mobile telephone;
- many others ...

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Signal  $f(t)$  is usually defined on the real line  $\mathbf{R}$ :



Typically, the origin at time  $t = 0$  is thought of as being *now*, with  $f(t)$  for  $t > 0$  being thought of as the *future* time evolution of the signal, and  $f(t)$  for  $t < 0$  being the *past* time evolution.

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## Types of signals

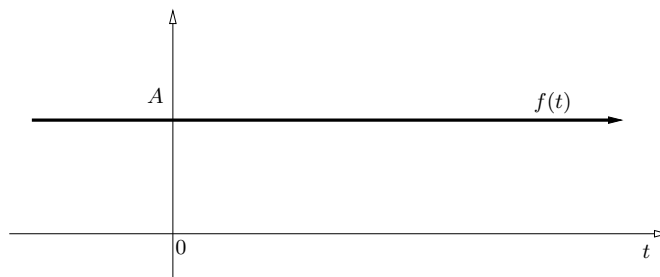
- There are many different kind of signals, with time evolutions qualitatively and quantitatively very different.
- It is useful to be able to classify signals considering simplified but common signal types.

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## Types of signals

- **Constant signal**

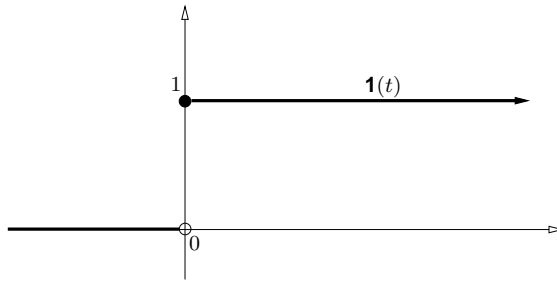
$$f(t) \triangleq A \in \mathbf{R}, \quad t \in (-\infty, \infty).$$



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### Step signal

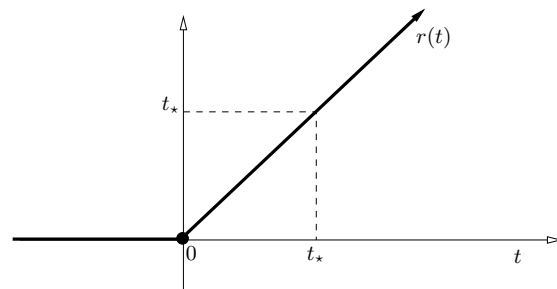
$$\mathbf{1}(t) \triangleq \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0. \end{cases}$$



## Types of signals

### Ramp signal

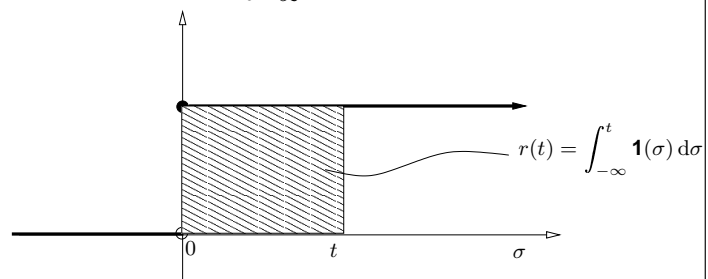
$$r(t) \triangleq \begin{cases} t & ; t \geq 0 \\ 0 & ; t < 0. \end{cases}$$



### Ramp signal

The ramp signal  $r(t)$  may be derived from another, since it is the cumulative area under the step signal  $\mathbf{1}(t)$  up until time  $t$ :

$$r(t) = \int_{-\infty}^t \mathbf{1}(\sigma) d\sigma.$$



As a consequence, and using the fundamental principle that differentiation is the inverse operation to integration, we also have that:

$$\frac{d}{dt}r(t) = \mathbf{1}(t).$$

That is, the time rate of change of the ramp signal is the step signal.

Finally,  $r(t)$  and  $\mathbf{1}(t)$  are also related by the equation

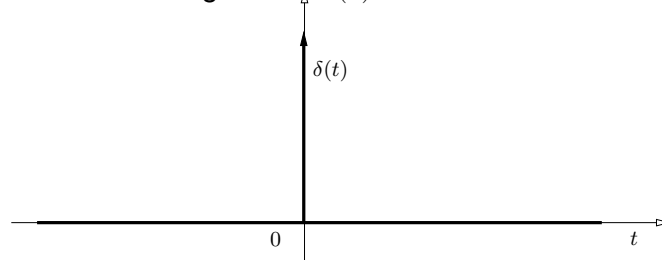
$$r(t) = t \cdot \mathbf{1}(t).$$

## Types of signals

- (Dirac) **Delta Function** or (Unit) **Impulse signal**

$$\delta(t) \triangleq \begin{cases} 0 & ; t \neq 0 \\ \text{Undefined} & ; t = 0 \end{cases}$$

It is useful to imagine that  $\delta(0) = +\infty$ :



## Impulse signal

*Dirac* refers to the Quantum Physics pioneer Paul Dirac who used this  $\delta$  to represent electrons as units of charge occupying an infinitesimal amount of space.

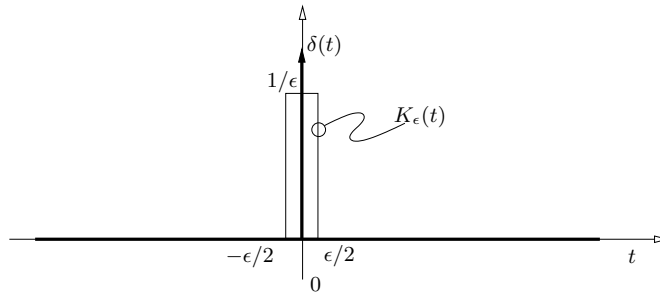
Strictly,  $\delta(t)$  is **not** a function. It may be characterised as a relationship for any continuous function  $f(t)$ :

$$\int_{-\infty}^{\infty} f(t) \delta(t) d\tau = f(0).$$

That is,  $\delta(t)$  *picks out* the value of  $f(t)$  at  $t = 0$ . This means that  $\delta(t)$  is *distribution* rather than a function.

$\delta(t)$  can be approximated by the function  $K_\epsilon(t)$  defined as:

$$\delta(t) \approx K_\epsilon(t) = \begin{cases} 1/\epsilon & ; |t| \leq \epsilon/2 \\ 0 & ; |t| > \epsilon/2 \end{cases}$$



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### Impulse signal

Hence, we have that:

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt \approx \int_{-\infty}^{\infty} f(t) K_\epsilon(t) dt = \int_{-\epsilon/2}^{\epsilon/2} f(t) \frac{1}{\epsilon} dt.$$

If  $\epsilon$  is made *small*, then  $f(t) \approx f(0)$  for all  $t \in (-\epsilon/2, \epsilon/2)$  since it is continuous. In this case:

$$\int_{-\epsilon/2}^{\epsilon/2} f(t) \frac{1}{\epsilon} dt \approx \frac{f(0)}{\epsilon} \int_{-\epsilon/2}^{\epsilon/2} dt = \frac{f(0)}{\epsilon} \cdot \epsilon = f(0).$$

And, when  $\epsilon \rightarrow 0$ :

$$\lim_{\epsilon \rightarrow 0} \int_{-\epsilon/2}^{\epsilon/2} f(t) \frac{1}{\epsilon} dt = f(0).$$

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### Impulse signal

Considering

$$\delta(t) = \lim_{\epsilon \rightarrow 0} K_\epsilon(t)$$

then

$$\begin{aligned} \int_{-\infty}^{\infty} f(t) \delta(t) dt &= \int_{-\infty}^{\infty} f(t) \lim_{\epsilon \rightarrow 0} K_\epsilon(t) dt \\ &= \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} f(t) K_\epsilon(t) dt = f(0). \end{aligned}$$

This is also why  $\delta(t)$  is known as the *unit impulse*, since:

$$\int_{-\infty}^{\infty} \delta(t) dt = \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} K_\epsilon(t) dt = \lim_{\epsilon \rightarrow 0} \int_{-\epsilon/2}^{\epsilon/2} \frac{1}{\epsilon} dt = 1.$$

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Finally, notice that there is an integral relationship between the unit step  $\mathbf{1}(t)$  and the Dirac delta  $\delta(t)$ :

$$\mathbf{1}(t) = \int_{-\infty}^t \delta(\sigma) d\sigma$$

And, by differentiating both sides, there is also a differential relationship:

$$\frac{d}{dt}\mathbf{1}(t) = \delta(t).$$

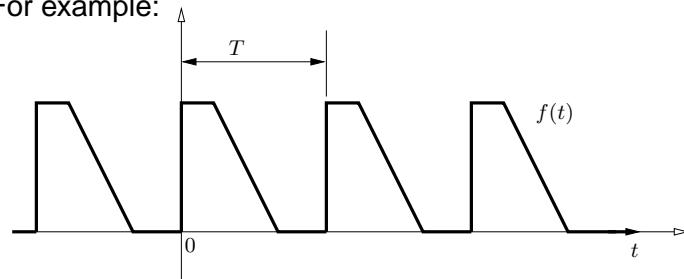
## Types of Signals

### • Periodic signal

It is a signal which repeats the same wave-shape every  $T$  seconds and infinitely often, i.e.:

$$f(t) = f(t + T), \quad \text{for all } t.$$

For example:



### Periodic signal

The signal

$$f(t) = \sin \omega t$$

is periodic with period  $T = 2\pi/\omega$ , since:

$$\begin{aligned} f(t + T) &= \sin \omega \left( t + \frac{2\pi}{\omega} \right) = \sin(\omega t + 2\pi) \\ &= \sin \omega t \underbrace{\cos 2\pi}_1 + \underbrace{\sin 2\pi}_0 \cos \omega t = \sin \omega t = f(t). \end{aligned}$$

Also, if  $f(t) = f(t + T)$  and  $g(t) = g(t + T)$  then for any  $\alpha, \beta \in \mathbf{R}$ :

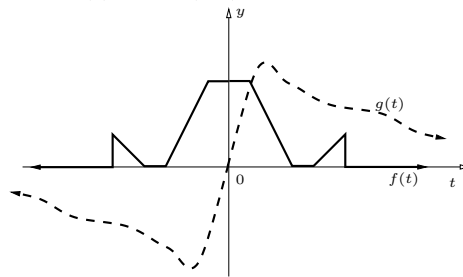
$$h(t) = \alpha f(t) + \beta g(t) = \alpha f(t + T) + \beta g(t + T) = h(t + T).$$

- **Even signal:** symmetric about the  $y$  axis.

$$f(t) = f(-t), \quad \text{for all } t.$$

- **Odd signal:** symmetric respect to the origin.

$$g(t) = -g(-t), \quad \text{for all } t.$$



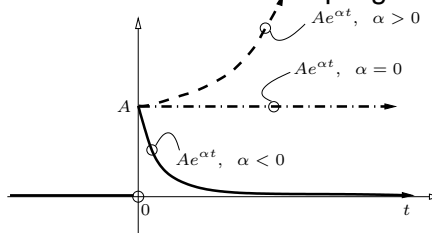
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## Types of Signals

- **Exponential signal**

$$f(t) = \mathbf{1}(t) \cdot Ae^{\alpha t} = \begin{cases} Ae^{\alpha t} & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

If  $\alpha > 0$  then  $e^{\alpha t}$  is exponentially increasing,  
if  $\alpha < 0$  then  $e^{\alpha t}$  is exponentially decreasing, and  
if  $\alpha = 0$  then  $e^{\alpha t} = 1$  and it is the step signal  $\mathbf{1}(t)$ .



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## Types of Signals

- **Exponential signals:** complex representation for signals with oscillating components.

$$f(t) = \mathbf{1}(t) \cdot Ae^{\alpha t} \cos(\omega t + \phi) = \begin{cases} Ae^{\alpha t} \cos(\omega t + \phi) & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

where:

- the oscillation period is  $T = 2\pi/\omega$ ,
- the exponential envelope is  $Ae^{\alpha t}$ , and
- the co-sinusoidal shape has phase offset  $\phi$ .

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It is useful to note that:

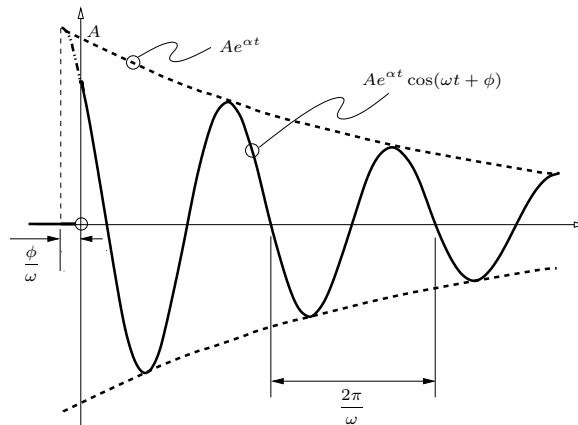
$$\begin{aligned} Ae^{\alpha t + j(\omega t + \phi)} &= Ae^{\alpha t} \cdot e^{j(\omega t + \phi)} \\ &= Ae^{\alpha t} [\cos(\omega t + \phi) + j \sin(\omega t + \phi)] \end{aligned}$$

Then

$$f(t) = Ae^{\alpha t} \cos(\omega t + \phi) = \mathbf{Real} \left\{ Ae^{\alpha + j(\omega t + \phi)} \right\}.$$

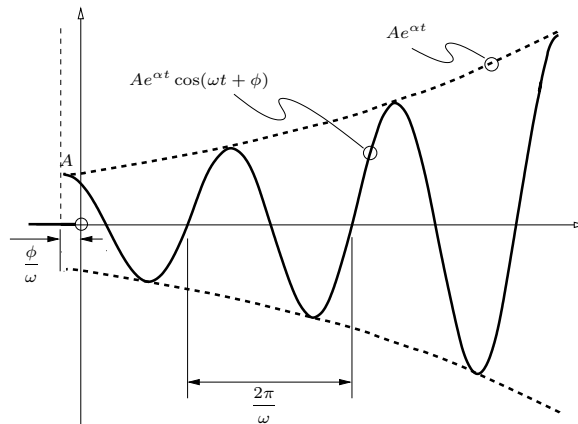
### Exponential signals (complex representation)

If  $\alpha < 0$ , then  $f(t) = Ae^{\alpha t} \cos(\omega t + \phi)$  is a **decreasing** oscillation:

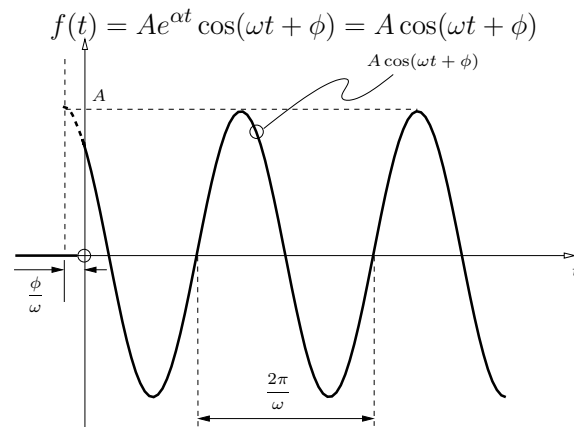


### Exponential signals (complex representation)

If  $\alpha > 0$ , then  $f(t) = Ae^{\alpha t} \cos(\omega t + \phi)$  is an **increasing** oscillation:



If  $\alpha = 0$ , then:

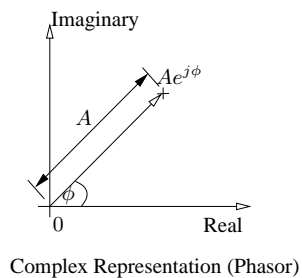


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## Exponential signals (complex representation)

For this *pure* sinusoidal signals, it is common to use a **phasor** representation:

$$f(t) = \text{Real} \left\{ Ae^{j(\omega t + \phi)} \right\} = A \cos(\omega t + \phi)$$



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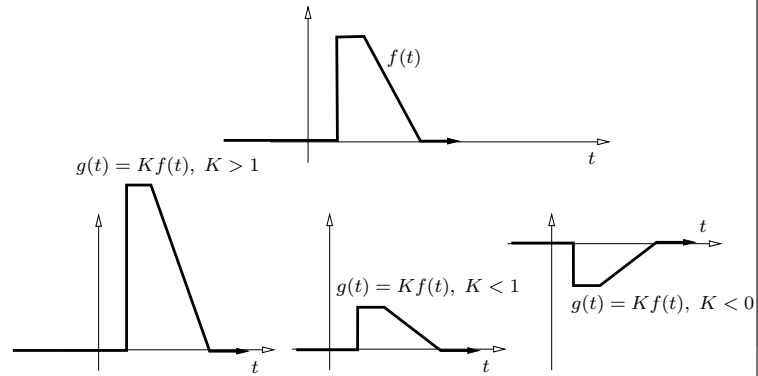
## Operations on Signals

- More complicated signals can be derived (or expressed) via various **fundamental operations** over the basic signals previously defined.

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- Magnitude Scaling:** It is the multiplication by a constant  $K \in \mathbb{R}$ :

$$g(t) = Kf(t).$$



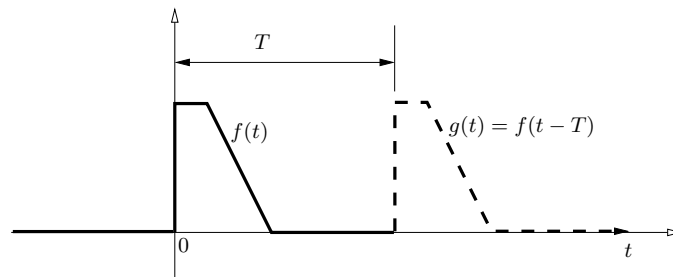
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## Operations on Signals

- Time Shifting (Translation):**

$$g(t) = f(t - T).$$

If  $T > 0$ , then  $g(t) = f(t - T)$  is  $f(t)$  *delayed* by  $T$  seconds:



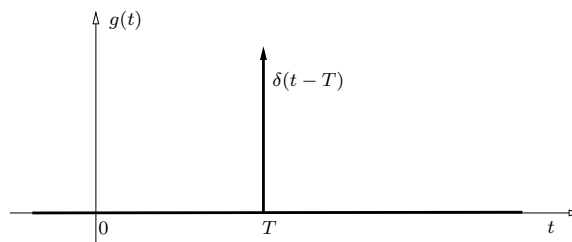
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## Time Shifting

Notice that, for Dirac delta signals:

$$g(t) = \delta(t - T) = \begin{cases} 0 & ; t \neq T \\ \text{Undefined} & ; t = T. \end{cases}$$

We can think of  $\delta(t - T)$  as having all its *mass* concentrated at  $t = T$ :



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And, for a continuous function  $f(t)$ , we have that:

$$\int_{-\infty}^{\infty} f(t)\delta(t - T) dt = f(T).$$

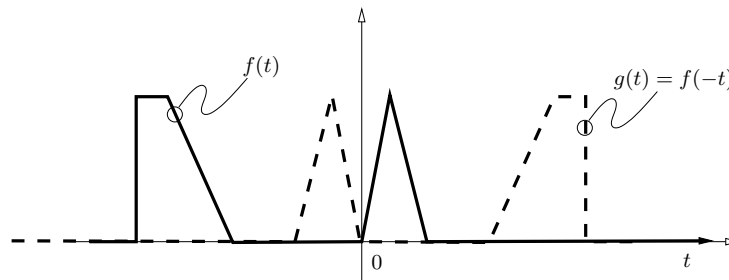
This principle will turn out to be of great importance in later developments.

## Operations on Signals

### Time Reversal (Flipping):

$$g(t) = f(-t)$$

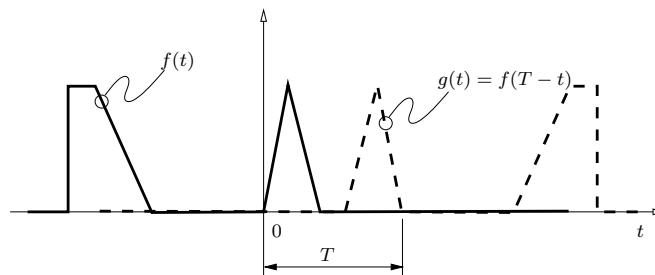
This operation produces a new signal  $g(t)$  by reversing the time direction:



### Time Reversal

In some situations time reversal operation is combined with the time shifting operation (e.g., in the *convolution*).

For example, consider  $g(t) = f(T - t)$ :

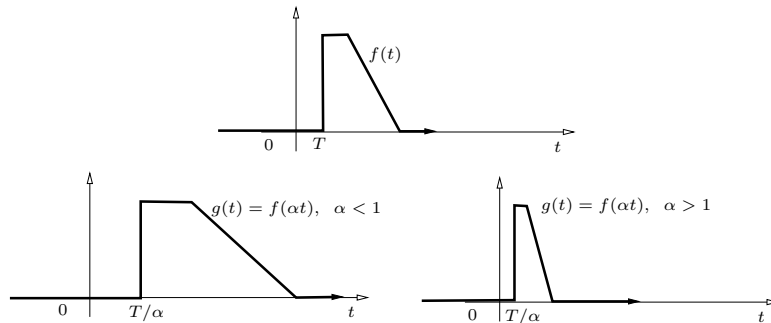


Note that, if  $h(t) = f(-t)$  (flipping)

then  $h(t - T) = f(-(t - T)) = f(T - t) = g(t)$  (shifting)

- **Time Scaling.** The signal is stretched or compressed by a factor  $\alpha \in \mathbf{R}^+$ :

$$g(t) = f(\alpha t)$$



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## Time Scaling

Note that:

- If  $\alpha < 0$ , there is a time reversal and scaling.
- For the Dirac delta function  $\delta(\alpha t)$ , using the change of variable  $\sigma = \alpha t$  which implies that  $d\sigma = \alpha dt$ , we have

$$\int_{-\infty}^{\infty} f(t)\delta(\alpha t) dt = \int_{-\infty}^{\infty} f\left(\frac{\sigma}{\alpha}\right)\delta(\sigma)\frac{1}{\alpha} d\sigma = \frac{1}{\alpha}f(0)$$

so that

$$\delta(\alpha t) = \frac{1}{\alpha}\delta(t).$$

i.e., the time scaling decrease or increase the *area* under the delta function.

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## Concluding Summary

We introduced some of the most fundamental ideas in the field of *Signals and Systems*:

- A signal is the time evolution of a quantity;
- It can be represented as a mathematical function  $f(t) : \mathbf{R} \rightarrow \mathbf{R}$ ;

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We introduced some of the most fundamental ideas in the field of *Signals and Systems*:

- There are some fundamental signal types,
  1. Constant:  $f(t) = K$ ;
  2. Unit step:  $f(t) = \mathbf{1}(t)$ ;
  3. Unit ramp:  $f(t) = r(t) = t \cdot \mathbf{1}(t)$ ;
  4. Dirac Delta:  $f(t) = \delta(t)$ ;
  5. Periodic of period  $T$ :  $f(t) = f(t + T)$ ;
  6. Even  $f(t) = f(-t)$  and odd  $f(t) = -f(-t)$ ;
  7. Exponential:  $f(t) = Ae^{\alpha t}$ ;
  8. Generalised Exponential:  $f(t) = Ae^{\alpha t} \cos(\omega t + \phi)$ .

## Concluding Summary

We introduced some of the most fundamental ideas in the field of *Signals and Systems*:

- There are some fundamental signal operations,
  1. Magnitude Scaling:  $g(t) = Kf(t)$ ;
  2. Time shifting (translation):  $g(t) = f(t - T)$ ;
  3. Time reversal (flipping):  $g(t) = f(-t)$ ;
  4. Time scaling:  $g(t) = f(\alpha t)$ .