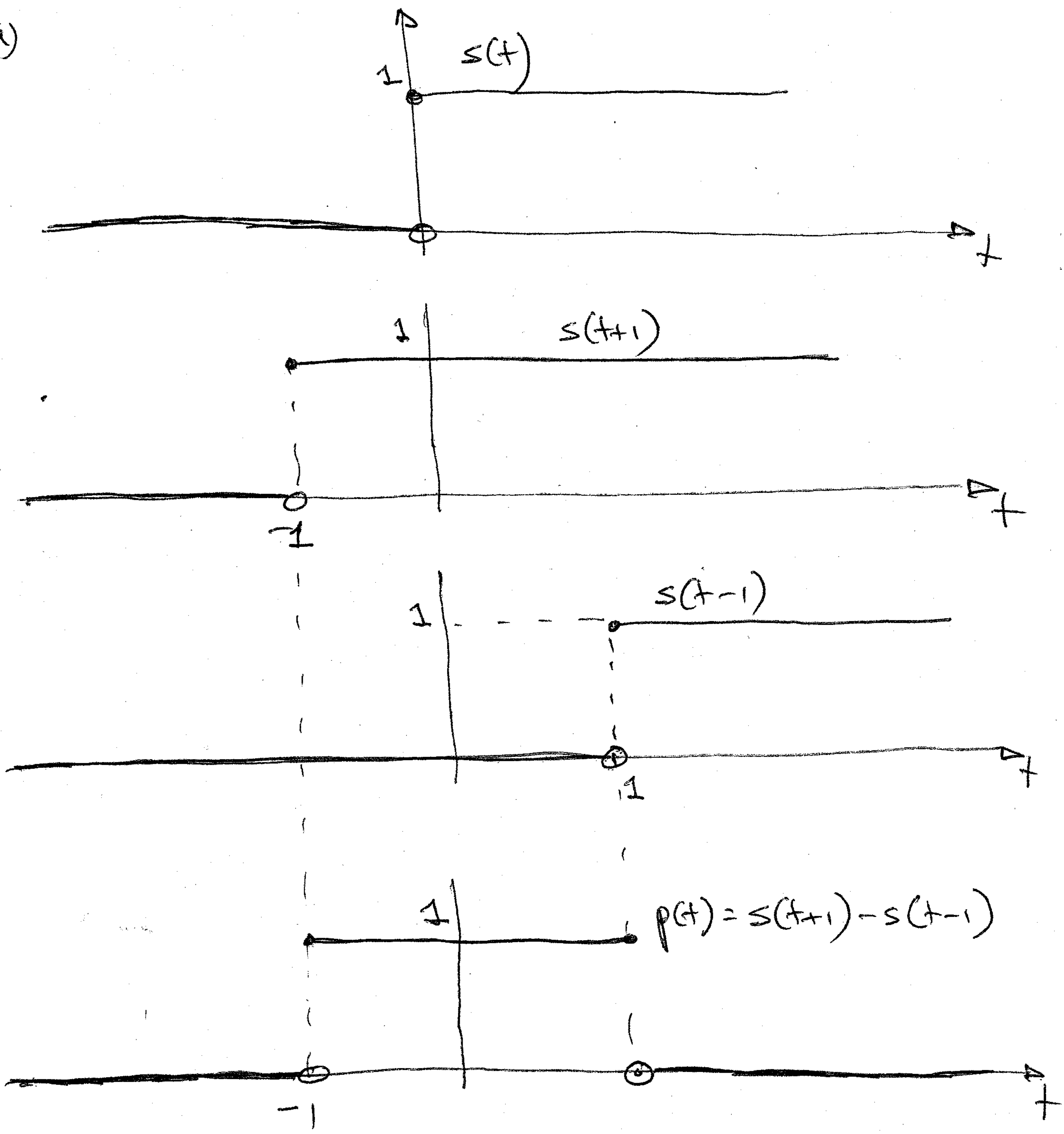
Tutorial #1 Solutions

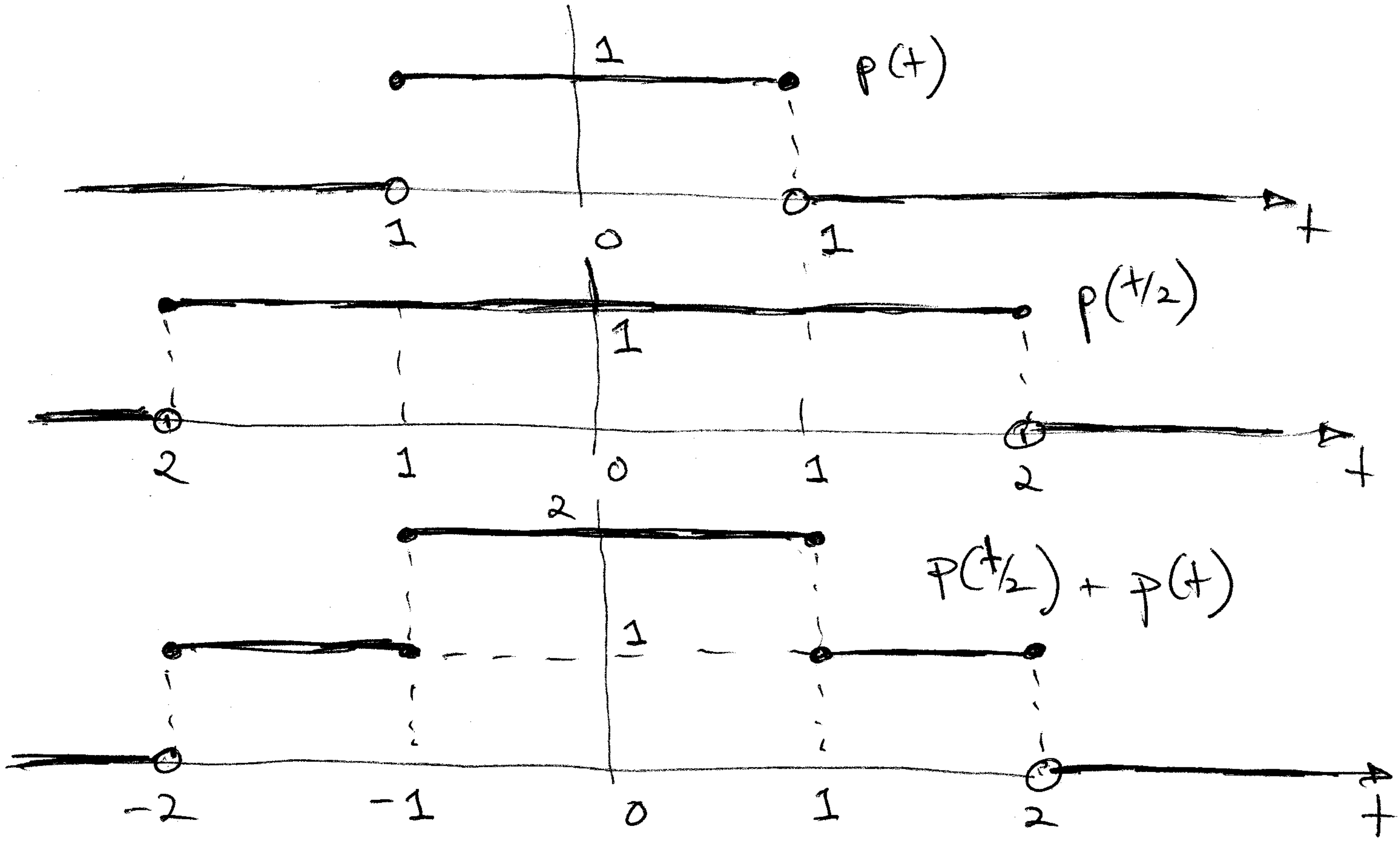
1/

(a)

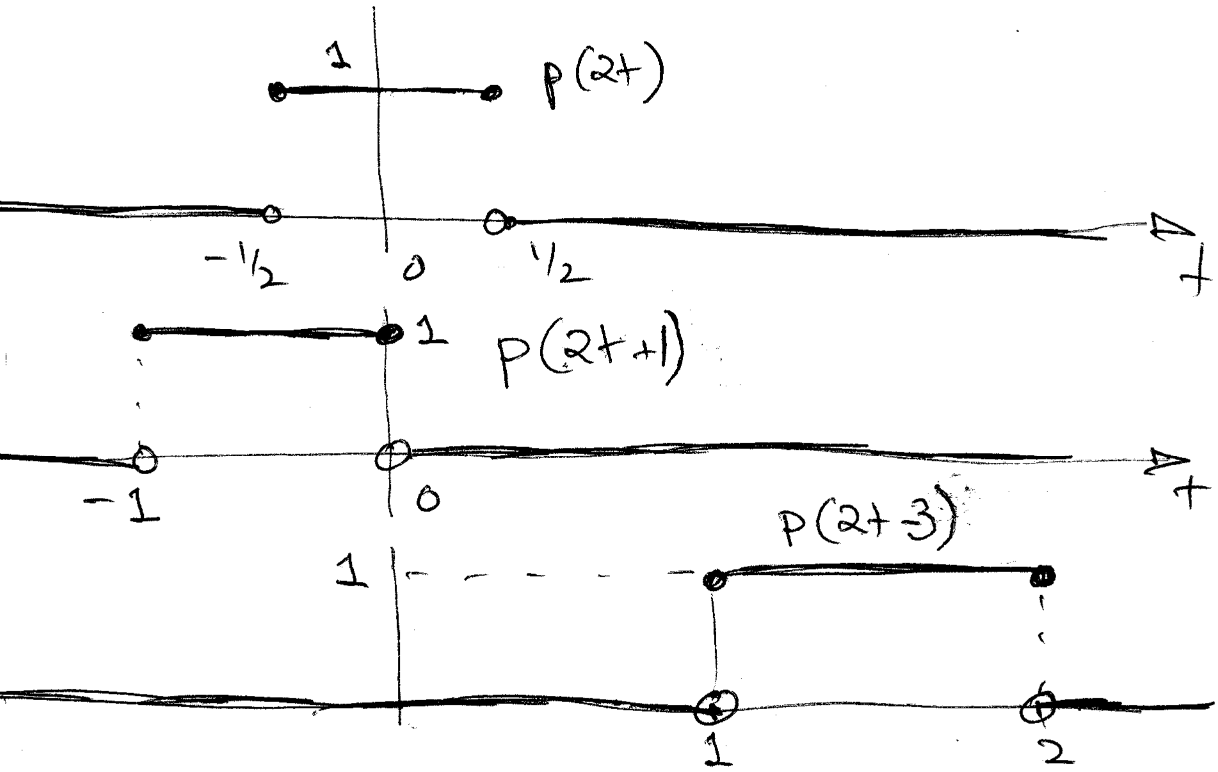


1/ (b) (i)

$$f(t) = p(t/2) + p(t)$$

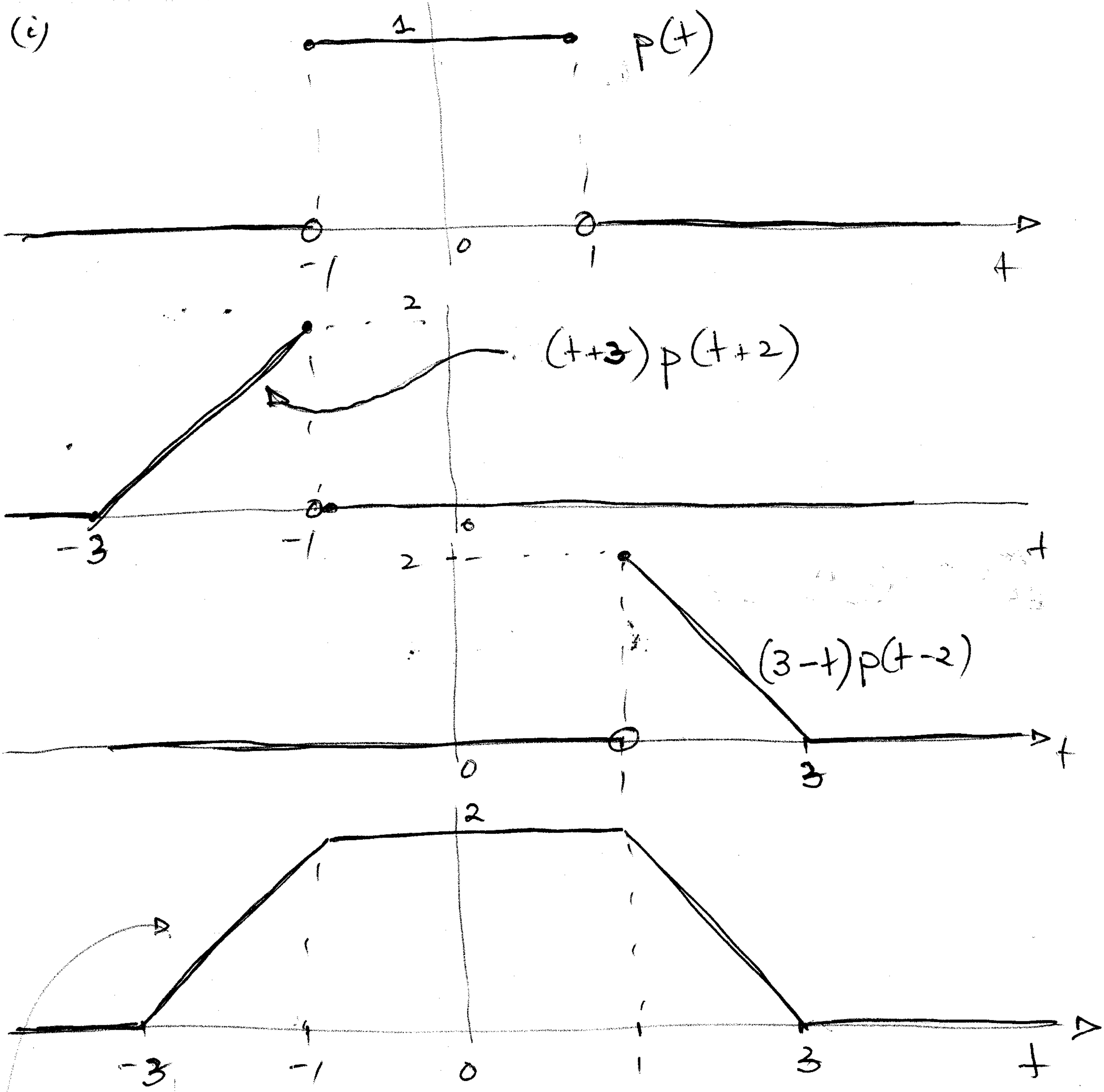


1/ (b) (ii)



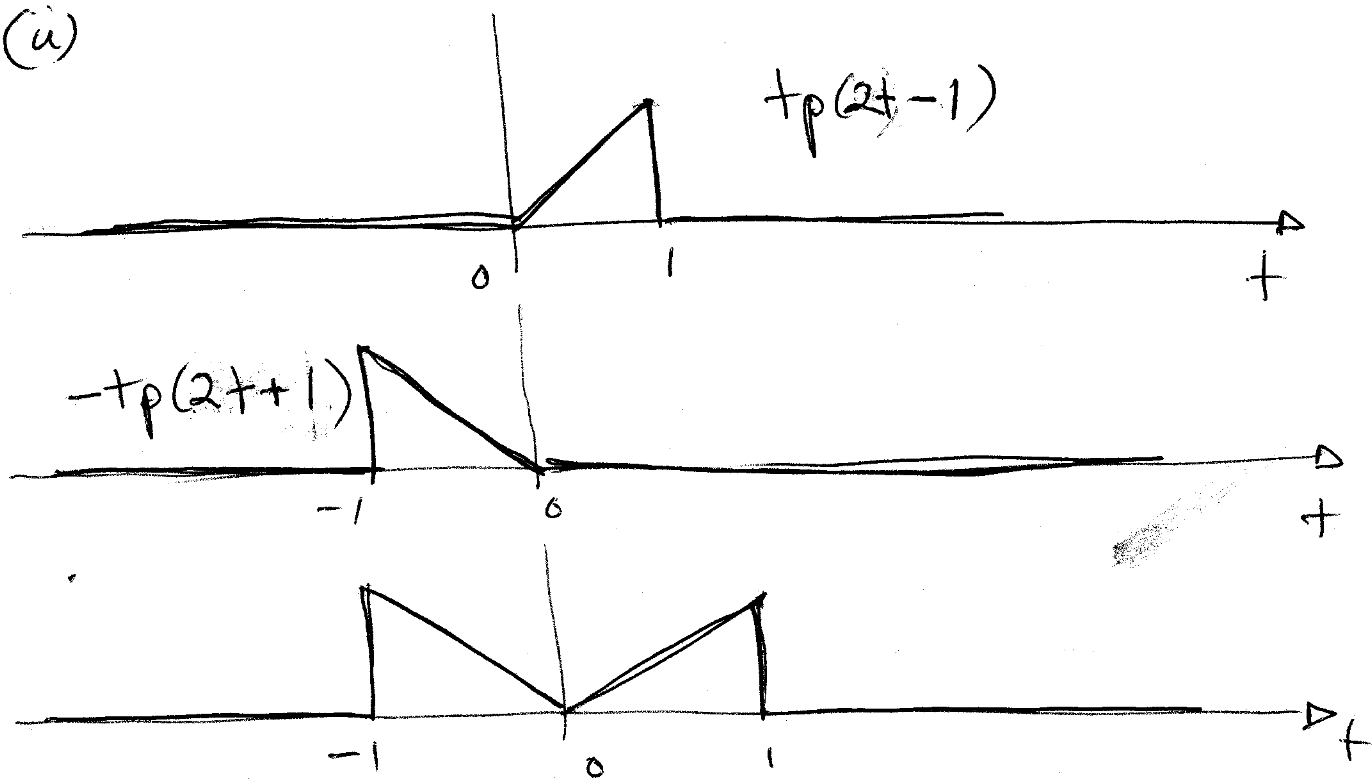
$$g(t) = \sum_{k=-\infty}^{\infty} p\left(2t + 4k + 1\right)$$

2/ (i)



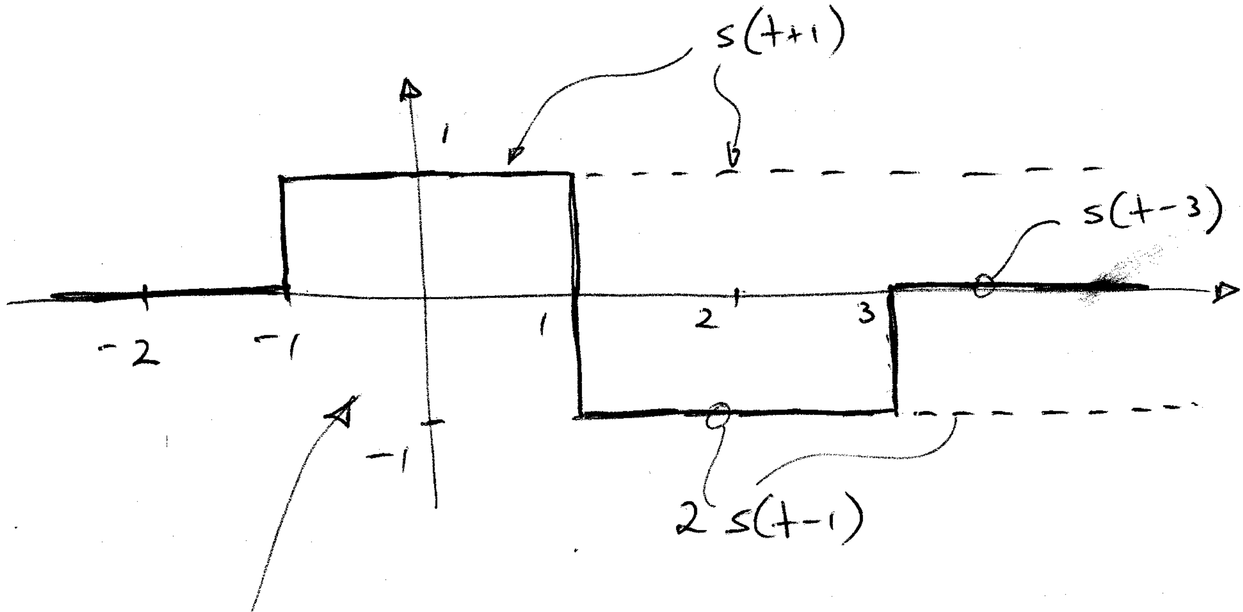
$$2p(t) + (t+3)\,p(t+2) + (3-t)\cdot p(t-2)$$

2/ (ii)



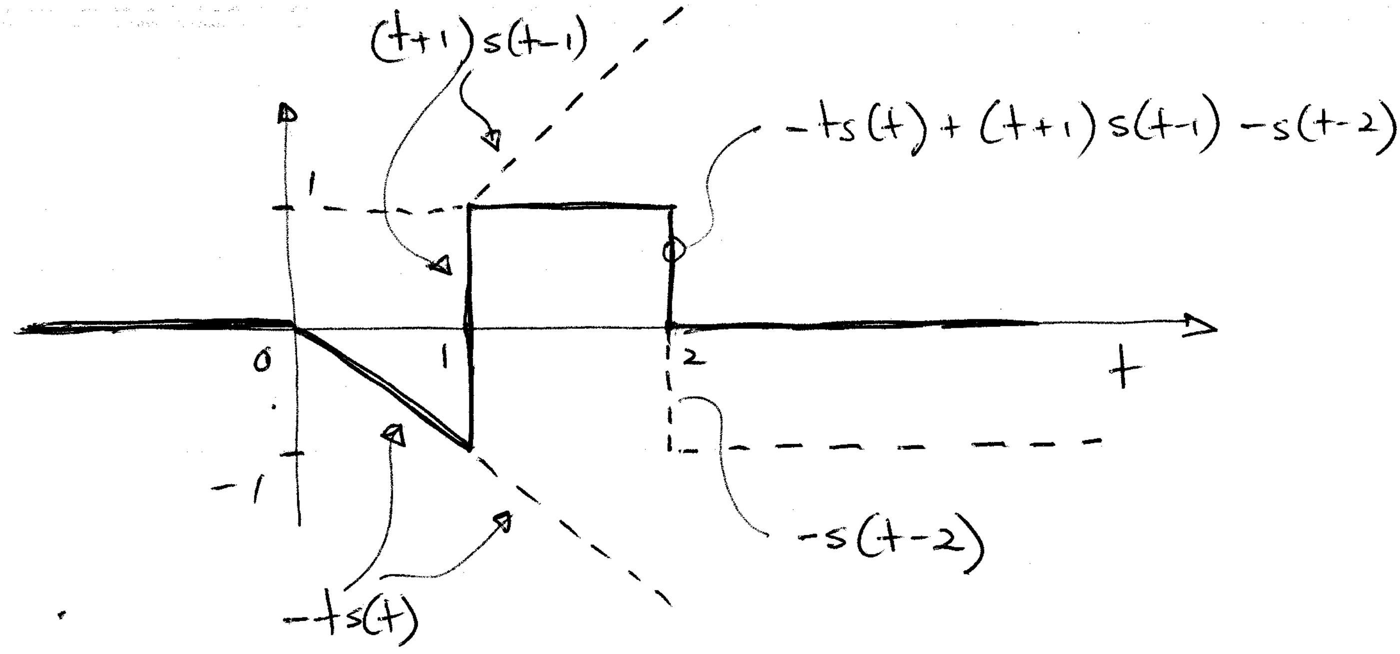
$tp(2t-1) - tp(2t+1)$

3/ (a)

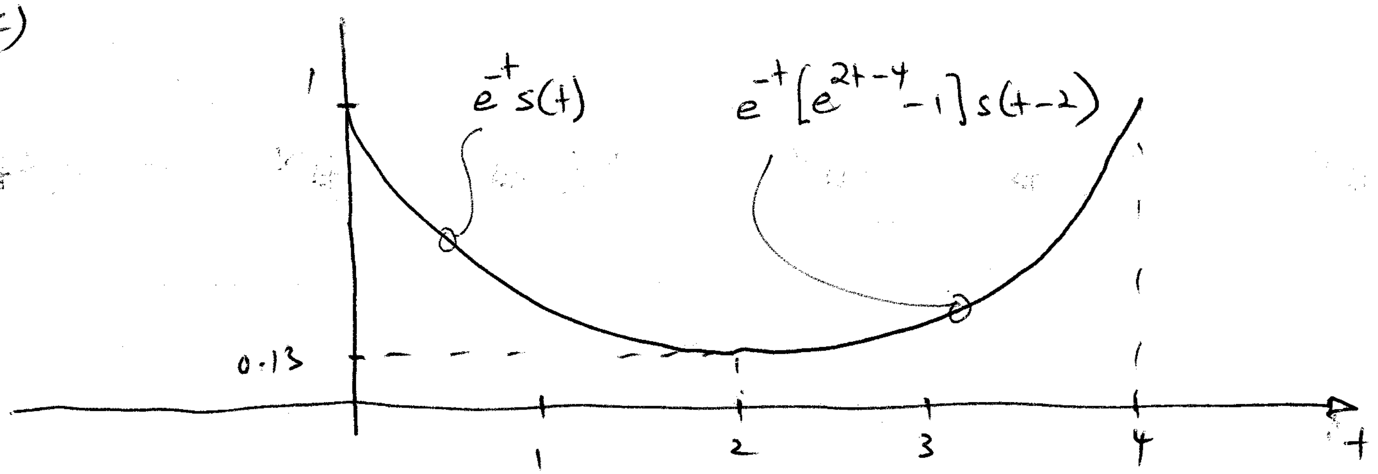


$s(t+1) - 2s(t-1) + s(t-3)$

3/ (b)

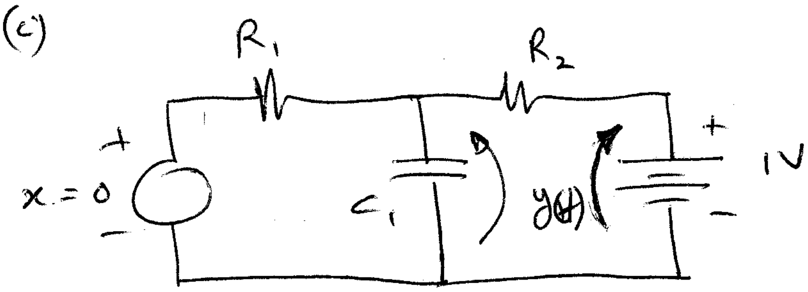


(c)



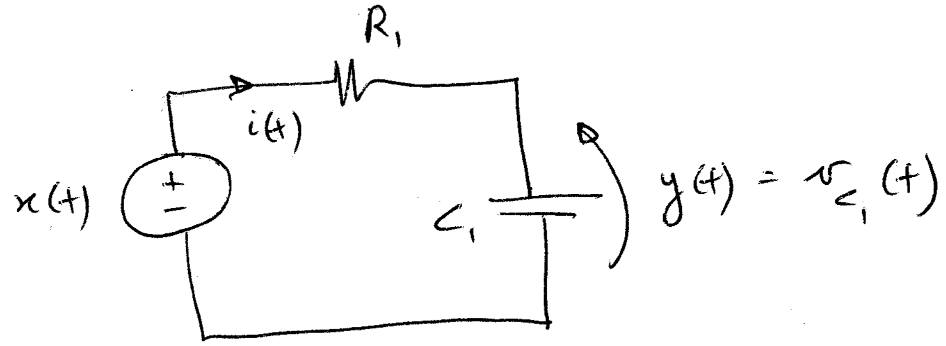
4/ For $t<0$ circuit is (switch is closed)

(c)



So $y(0) = v_{c_1} = \dfrac{1 \cdot R_1}{R_1 + R_2}$ (voltage division)

4/(ii) For $t \geq 0$ switch is open and circuit becomes



By KVL: $x(t) = R_1 i(t) + y(t)$ (*)

By law of capacitive action

$$i(t) = C_1 \frac{dy(t)}{dt} \qquad (**)$$

Substituting (**) in (*) then gives the d.e.

$$x(t) = R_1 C_1 \frac{dy(t)}{dt} + y(t)$$

or

$$\frac{dy(t)}{dt} + \frac{1}{R_1 C_1} y(t) = \frac{1}{R_1 C_1} x(t) \qquad (***)$$

(iii) Using integrating factor the de. (***) can be rewritten as

$$\frac{d}{dt}\left[y(t)\, e^{t/R_1 C_1} \right] = \frac{e^{t/R_1 C_1}}{R_1 C_1} x(t)$$

Integrating both sides from $t = 0$ to $t = t$

$$y(t)e^{t/R_1C_1} - y(0) = \frac{1}{R_1C_1}\int_0^t e^{\sigma/R_1C_1}x(\sigma)\,d\sigma$$

However $x(t) = 0$ so the RHS above $= 0$ and therefore

$$y(t) = y(0)e^{-t/R_1C_1} = \frac{R_1}{R_1+R_2}e^{-t/R_1C_1} \quad ; t \geq 0$$

5/(a) By KVL

$$x(t) = Ri(t) + y(t)$$

Differentiating both sides ($\frac{d}{dt}$ this) gives

$$\frac{dx(t)}{dt} = R\frac{d}{dt}i(t) + \frac{d}{dt}y(t)$$

By Law for inductor;

$$y(t) = L\frac{d}{dt}i(t)$$

Therefore

$$\frac{dx(t)}{dt} = \frac{d}{dt}y(t) + \frac{R}{L}y(t)$$

(b) Use integrating factor $e^{R/L\,t}$ to re-write d.e. as

$$\frac{d}{dt}\left[e^{R/L\,t}\, y(t) \right] = e^{R/L\,t} \frac{d\,x(t)}{dt}$$

Integrating both sides from 0 to t then gives

$$y(t)\, e^{R/L\,t} - y(0) = \int_0^t e^{R/L\,\sigma} \frac{d\,x(\sigma)}{d\sigma}\, d\sigma$$

Now when $x(t)$ is a unit step, $\frac{d}{dt} x(t) = \delta(\sigma)$

so the RHS above is

$$\int_0^t e^{R/L\,\sigma} \frac{d\,x(\sigma)}{d\sigma}\, d\sigma = \int_0^t e^{R/L\,\sigma}\, \delta(\sigma)\, d\sigma = 1$$

and therefore

$$y(t) = y(0)\, e^{-R/L\,t} + e^{-R/L\,t}$$

If $i(t) = 0$ for $t < 0$ then $v_R = 0$ and hence $y(0) = x(0) = 0$ so that

$$y(t) = e^{-R/L\,t} \quad ; \quad t > 0$$