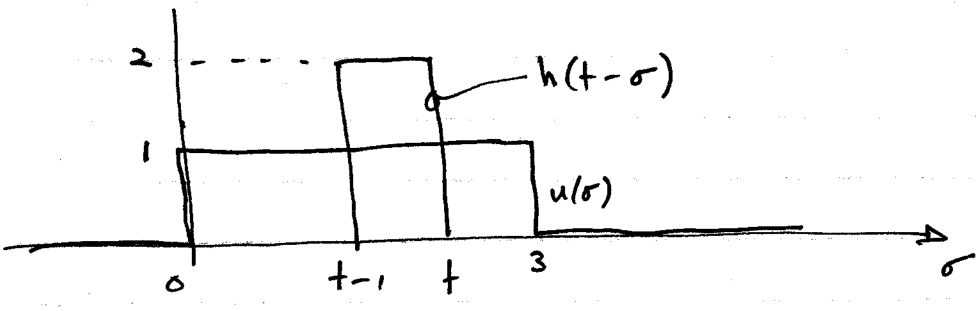
ELEC 240 Tutorial #3 Solutions

1/

(a)



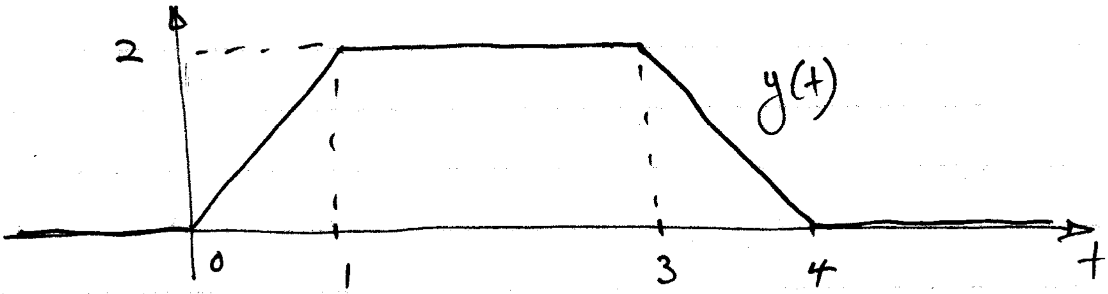
$$y(t) = \int_{-\infty}^{\infty} h(t-\sigma)\,u(\sigma)\,d\sigma$$

$$= 0 \quad ; \; t < 0$$

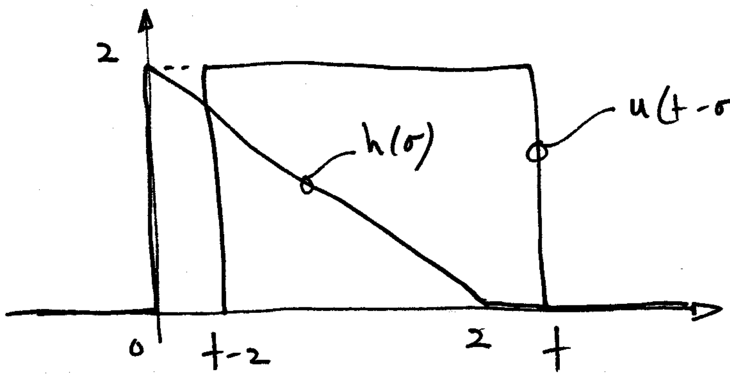
$$= \int_{0}^{t} 2.1\,d\sigma = 2\sigma \Big|_{\sigma=0}^{\sigma=t} = 2t \quad ; \; t \in (0,1)$$

$$= \int_{t-1}^{t} 2.1\,d\sigma = 2\sigma \Big|_{\sigma=t-1}^{\sigma=t} = 2t - 2(t-1) = 2 \; ; \; t \in (1,3)$$

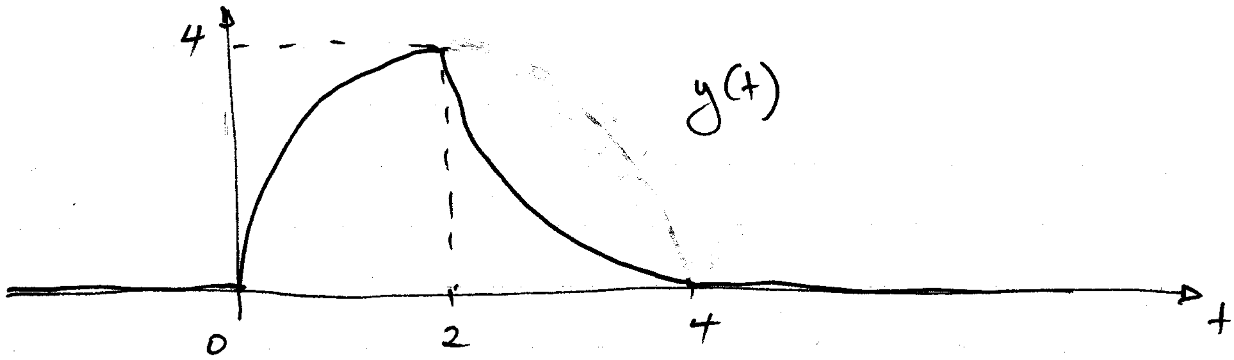
$$= \int_{t-1}^{3} 2.1\,d\sigma = 2\sigma \Big|_{\sigma=t-1}^{\sigma=3} = 6 - 2(t-1) = 8 - 2t \; ; \; t \in (3,4)$$



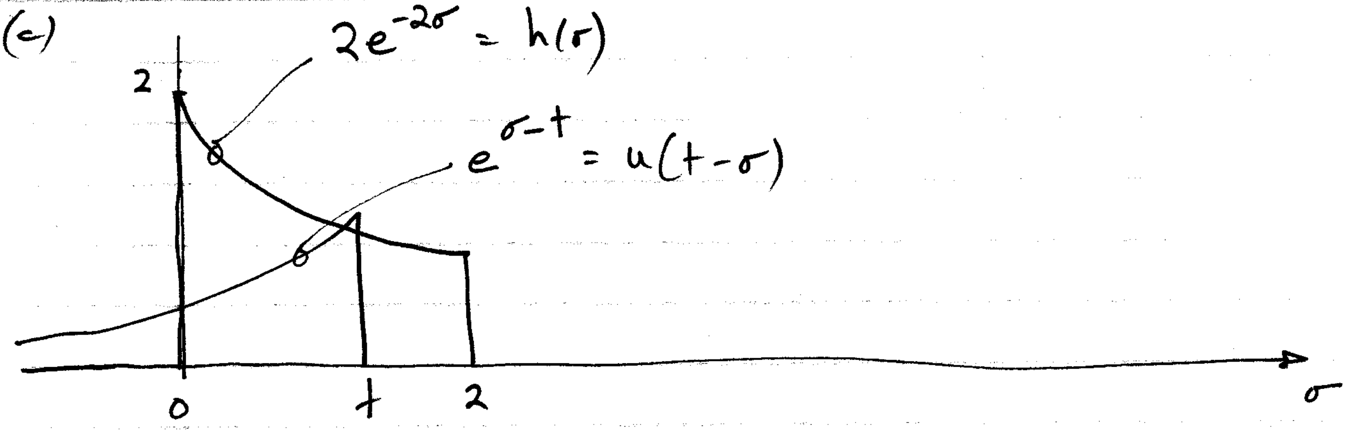
(b)



$$y(t) = \begin{cases} 0 & ; \ t < 0 \\ \displaystyle\int_0^t 2(2-\sigma)\,d\sigma = 4\sigma - \sigma^2 \Big|_0^t = 4t - t^2 & ; \ t \in [0,2] \\ \displaystyle\int_{t-2}^2 2(2-\sigma)\,d\sigma = 4\sigma - \sigma^2 \Big|_{t-2}^{2} = 16 - 8t + t^2 & ; \ t \in [2,4] \\ 0 & ; \ t > 4 \end{cases}$$



(c)



$$y(t) = 0 \quad ; \; t < 0$$

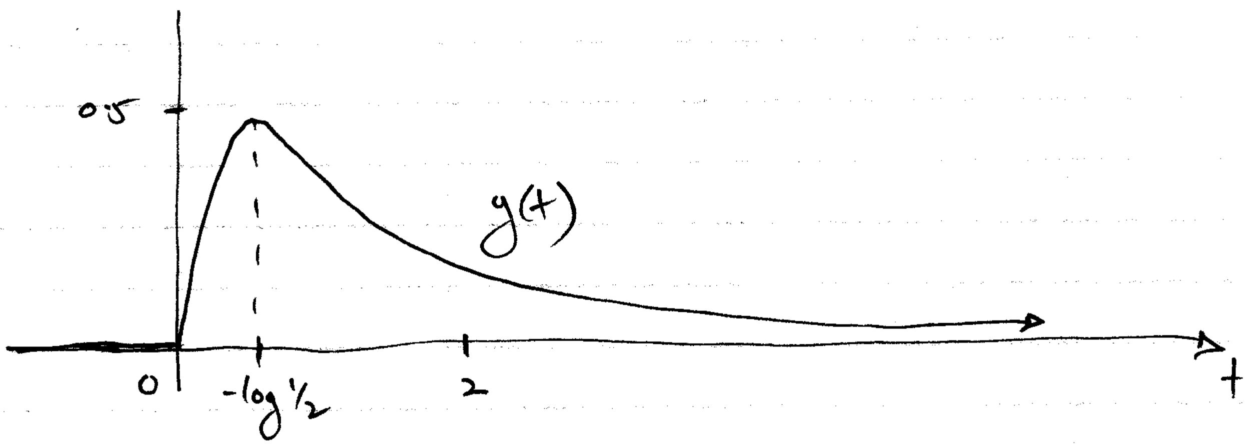
$$= \int_0^t 2e^{-2\sigma} e^{\sigma - t} \, d\sigma = 2e^{-t} \int_0^t e^{-\sigma} \, d\sigma$$

$$= 2e^{-t} \times \left. \frac{e^{-\sigma}}{-1} \right|_{\sigma=0}^{\sigma=t}$$

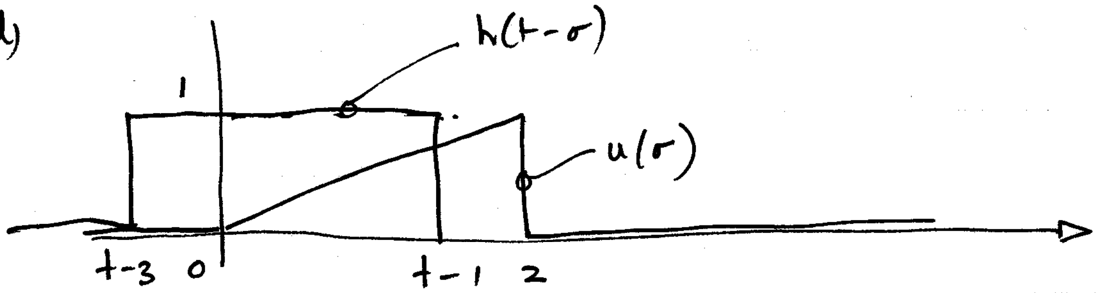
$$= 2e^{-t}\left[1 - e^{-t}\right] \quad ; \; t \in (0,2)$$

$$= \int_0^2 2e^{-2\sigma} e^{\sigma - t} = 2e^{-t} \times \left. \frac{e^{-\sigma}}{-1} \right|_{\sigma=0}^{\sigma=2}$$

$$= 2\left[1 - e^{-2}\right] e^{-t} \quad ; \; t \geq 2$$



(d)

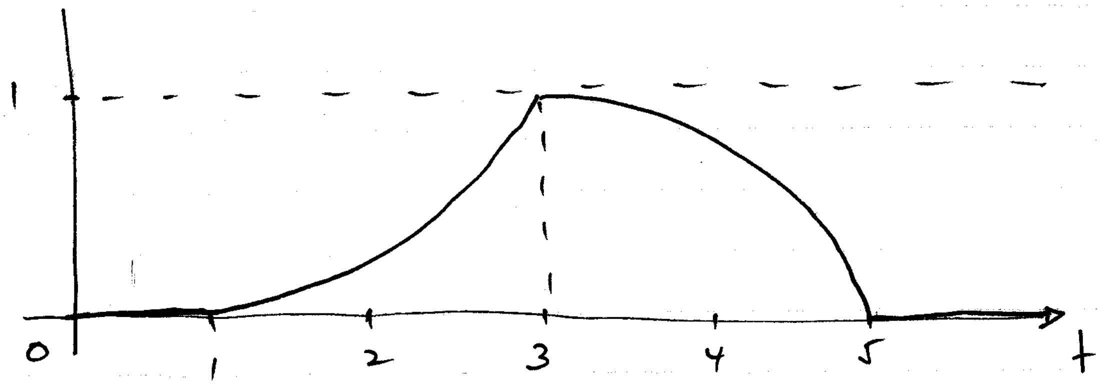


$$y(t) = 0 \quad ; \; t < 1$$

$$= \int_0^{t-1} \frac{\sigma}{2}\, d\sigma = \left.\frac{\sigma^2}{4}\right|_0^{t-1} = \frac{1}{4}(t-1)^2 \quad ; \; t \in [1, 3)$$

$$= \int_{t-3}^{2} \frac{\sigma}{2}\, d\sigma = \left.\frac{\sigma^2}{4}\right|_{t-3}^{2} = 1 - \frac{1}{4}(t-3)^2 \quad ; \; t \in [3, 5]$$

$$= 0 \quad ; \; t > 5$$



2/

(a)

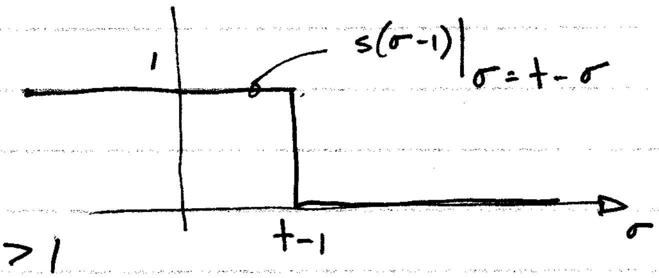
$$y(t) = 0 \quad ; \quad t < 0$$

$$= \int_0^t \left[e^{-\sigma} + \sin\sigma \right] d\sigma \quad ; \quad t > 0$$

$$= -e^{-\sigma} - \cos\sigma \Big|_{\sigma=0}^{\sigma=t} = 2 - e^{-t} - \cos t \quad ; \quad t > 0$$

(b) Response to $s(t)$ is as above, Response to $s(t-1)$ is

$$y(t) = 0 \quad ; \quad t < 1$$



$$= \int_0^{t-1} \left[e^{-\sigma} + \sin\sigma \right] d\sigma \quad ; \quad t > 1$$

$$= e^{-\sigma} - \cos\sigma \Big|_{\sigma=0}^{\sigma=t-1}$$

$$= 2 - e^{-(t-1)} - \cos(t-1)$$

So total response to $s(t) - s(t-1)$ is

$$y(t) = \left[2 - e^{-t} - \cos t\right] s(t) - \left[2 - e^{-(t-1)} - \cos(t-1)\right] s(t-1)$$

3/

(a) Impulse response is

$$h(t) = 0 \quad ; \quad t \le 0$$

$$= \int_{-\infty}^{t} (t - \sigma + 2)\, \delta(\sigma)\, d\sigma \quad ; \quad t > 0$$

$$= t + 2$$

(b) Response to $s(t)$ alone is

$$y(t) = 0 \quad ; \quad t < 0$$

$$= \int_{0}^{t} (\sigma + 2)\, d\sigma \quad ; \quad t \ge 0$$

$$= \left. \frac{\sigma^2}{2} + 2\sigma \right|_{\sigma=0}^{\sigma=t} = 2t + \frac{t^2}{2}$$

Therefore total response is

$$y(t) = 0 \quad ; \quad t < 0$$

$$= \left(2t + \frac{t^2}{2}\right) s(t) - 2\left(2(t-1) + \frac{(t-1)^2}{2}\right) s(t-1)$$

$$+ \left[2(t-2) + \frac{(t-2)^2}{2}\right] s(t-2)$$

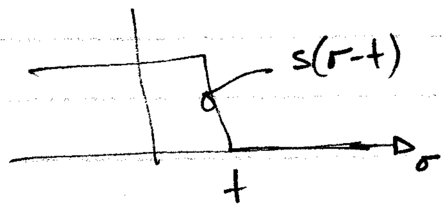
4/ Firstly, if $u(t) = \begin{cases} 1 - t & ; t \in [0,1] \\ 0 & ; \text{Otherwise} \end{cases}$

Then

$$z(t) = \frac{du(t)}{dt} = \begin{cases} \delta(t) - s(t) & ; t \in [0,1] \\ 0 & ; \text{Otherwise.} \end{cases}$$

Therefore, response of System 2 will comprise impulse & step response components. Step response component is given as

$$y(t) = 0.5\left(1 - e^{-2t}\right) s(t)$$



However, by convolution relationship, step response is

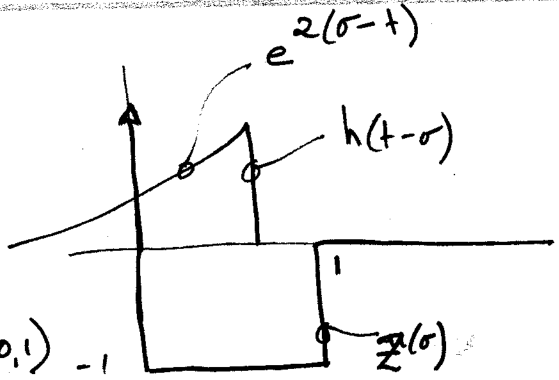
$$y_s(t) = \int_{-\infty}^{\infty} h(\sigma)\, s(t-\sigma)\, d\sigma$$

$$= \int_0^t h(\sigma)\, d\sigma$$

$$\Rightarrow \frac{d}{dt} y_s(t) = \frac{d}{dt}\int_0^t h(\sigma)\, d\sigma = h(t)$$

So impulse response is $h(t) = \frac{d}{dt}\, 0.5\left(1 - e^{-2t}\right) = e^{-2t}$

Therefore



$$z(t) = 0 \quad ; \; t < 0$$

$$= \int_0^t \underbrace{[\delta(\sigma)-1]}_{z(\sigma)} \underbrace{e^{2(\sigma-t)}}_{h(t-\sigma)} \, d\sigma \quad ; \; t \in [0,1)$$

$t \in [0,1)$

$$= e^{-2t} - e^{-2t} \cdot \frac{e^{2\sigma}}{2} \Bigg|_{\sigma=0}^{\sigma=t}$$

$$= e^{-2t} - \frac{e^{-2t}}{2}\left[e^{2t} - 1\right] = \frac{1}{2}\left[3e^{-2t} - 1\right]$$

$$= \int_0^1 [\delta(\sigma)-1] \, e^{2(\sigma-t)} \, d\sigma \quad ; \; t > 1$$

$t \geq 1$

$$= e^{-2t} - e^{-2t} \frac{e^{2\sigma}}{2} \Bigg|_{\sigma=0}^{\sigma=1}$$

$$= \left[\frac{3 - e^2}{2}\right] e^{-2t}$$