ELEC240 - Tutorial # 2

1. Write the following differential equations in state space form. Use both the observer and controller canonical forms.

   (a)
   $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 5\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 3y(t) = 4\frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t).$$

   (b)
   $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 5\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 3y(t) = 3\frac{\mathrm{d}^2}{\mathrm{d}t^2}u(t) + 4\frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t).$$

   (c)
   $$\frac{\mathrm{d}^4}{\mathrm{d}t^4}y(t) + \frac{\mathrm{d}^3}{\mathrm{d}t^3}y(t) + 2\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 5\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 3y(t) = 4\frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t).$$

   (d)
   $$\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 2y(t) = u(t).$$

2. Recall from lectures that the state-space system

   $$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)\\ y(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}$$

   has solution

   $$y(t) = \mathbf{C}e^{\mathbf{A}(t-t_0)}\mathbf{x}_0 + \int_{t_0}^{t} g(t-\sigma)u(\sigma)\,\mathrm{d}\sigma, \qquad g(t) \triangleq \mathbf{C}e^{\mathbf{A}t}\mathbf{B}$$

   and if

   $$\mathbf{A} = \begin{bmatrix} -(\alpha+\beta) & 1 \\ -\alpha\beta & 0 \end{bmatrix}$$

   then

   $$e^{\mathbf{A}t} = \frac{1}{(\alpha-\beta)}\begin{bmatrix} \alpha e^{-\alpha t} - \beta e^{-\beta t} & e^{-\beta t} - e^{-\alpha t} \\ \alpha\beta\left[e^{-\alpha t} - e^{-\beta t}\right] & \alpha e^{-\beta t} - \beta e^{-\alpha t} \end{bmatrix}.$$

   Use these facts to solve the following differential equation

   $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 8\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 15y(t) = u(t), \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right|_{t=0} = 1, \;\; y(0) = -0.5$$

   when $u(t)$ is the signal $u(t) = s(t-1)$ with $s(t)$ being the unit step. Plot the solution.

3. Compute and sketch the impulse response $h(t)$ associated with the following differential equation relationships

   (a)
   $$\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 4y(t) = u(t)$$

   (b)
   $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 8\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 15y(t) = u(t)$$

   (c)
   $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 5\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 6y(t) = \frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t)$$