ELEC240 - Tutorial # 3

1. Compute the convolution

$$y(t) = [h \circledast u](t) = \int_{-\infty}^{\infty} h(t-\sigma)u(\sigma)\,\mathrm{d}\sigma$$

for the following four cases of h and u shown in figure 1. Sketch the signals to be convolved as well as the result $y(t)$.

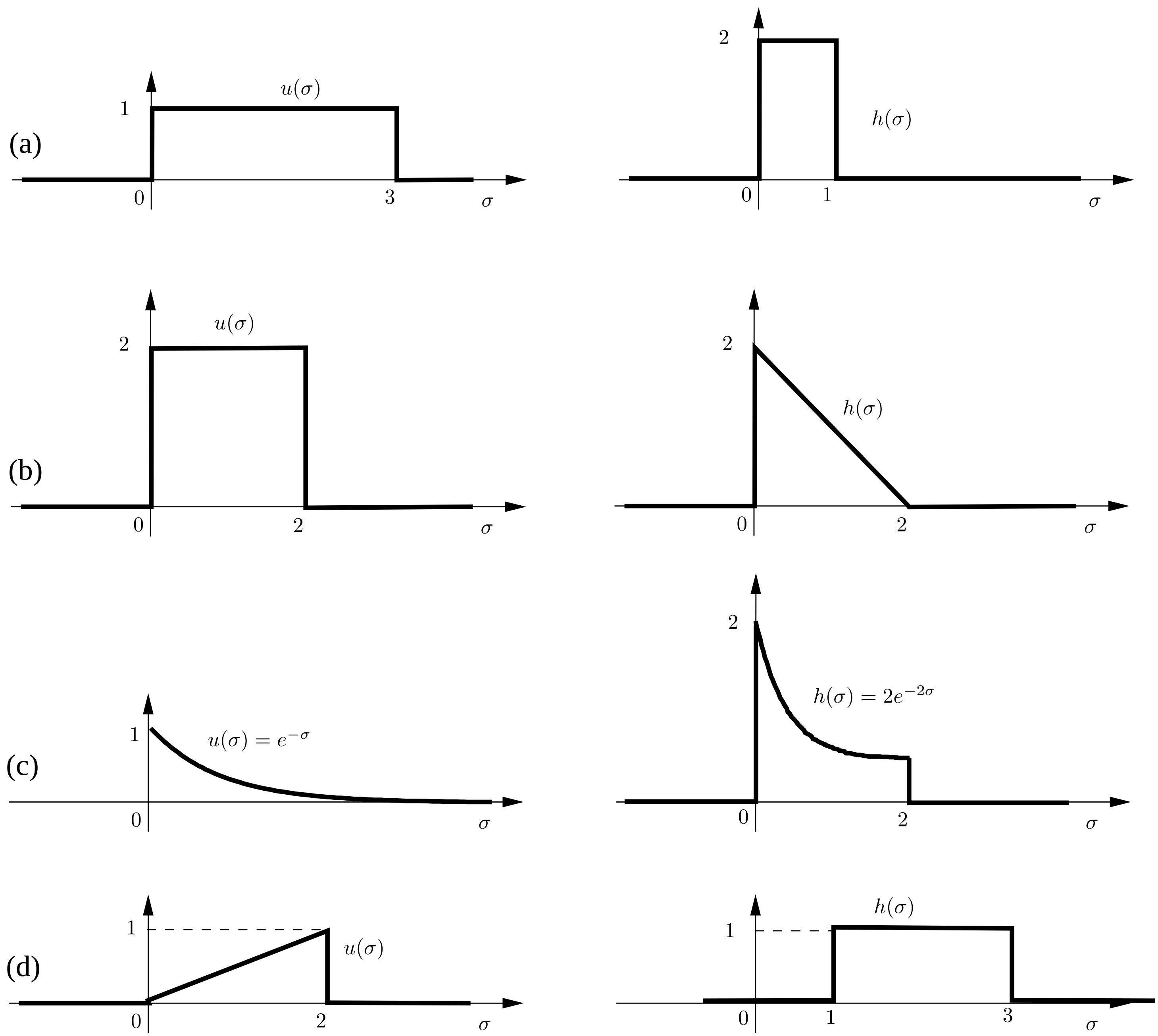

Figure 1: *Signals to be convolved in question 1.*

2. A linear and time invariant system has impulse response

$$h(t) = \begin{cases} e^{-t} + \sin t & ; t \geq 0 \\ 0 & ; t < 0. \end{cases}$$

(a) Compute the response of this system to an input which is the unit step; $u(t) = s(t)$.

(b) Compute the response of this system for the input $u(t) = s(t) - s(t-1)$.

3. A linear time-invariant system has input/output relationship

$$y(t) = \int_{-\infty}^{t} (t - \sigma + 2)u(\sigma)\,\mathrm{d}\sigma.$$

(a) Determine the impulse response $h(t)$ of this system.

(b) Compute the output response of this system when

$$u(t) = s(t) - 2s(t-1) + s(t-2)$$

and $s(t)$ is the unit step.

4. (Harder)

Consider the cascade connection shown in figure 2. System 1 obeys the differential equation relationship

$$z(t) = \frac{\mathrm{d}}{\mathrm{d}t}u(t).$$

Furthermore, it is known that system 2 is linear and time invariant and when its input $z(t)$ is the unit step $s(t)$ then the resulting output response is

$$y(t) = 0.5(1 - e^{-2t})s(t).$$

Now suppose that the input $u(t)$ to this cascade connection is

$$u(t) = \begin{cases} 1 - t & ; t \in [0, 1] \\ 0 & ; \text{Otherwise}. \end{cases}$$

Compute the resulting output response $y(t)$ of the cascade connection

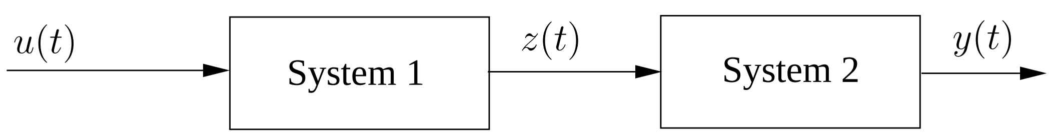

Figure 2: *System considered in question 4.*