ELEC240 - Tutorial # 4

1. Plot the following signals and compute their Laplace Transforms

   (a)
$$f(t) = \begin{cases} 0 & ; t < 0 \\ t & ; t \in [0,1) \\ 1 & ; t \in [1,3) \\ 4-t & ; t \in [3,4) \\ 0 & ; t \geq 4 \end{cases}$$

   (b)
$$f(t) = \begin{cases} 0 & ; t < 0 \\ e^{-t} & ; t \in [0,2) \\ e^{t-4} & ; t \in [2,4) \\ 0 & ; t \geq 4 \end{cases}$$

   (c)
$$f(t) = \begin{cases} 0 & ; t < 0 \\ \sin t & ; t \in [0,\pi) \\ -\sin t & ; t \in [\pi,2\pi) \\ 0 & ; t \geq 2\pi \end{cases}$$

2. A signal $f(t)$ has Laplace Transform
$$F(s) = \frac{s+1}{s^2+5s+7}.$$
   Using this information and the fundamental properties of the Laplace Transform, compute the transforms of the following signals

   (a)
$$g(t) = f(3t-4)$$

   (b)
$$g(t) = t\,f(t)$$

   (c)
$$g(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2} f(t)$$

   (d)
$$g(t) = \int_{-\infty}^{t} f(\sigma)\,\mathrm{d}\sigma$$

   (e)
$$g(t) = \sin 2t\, f(t)$$

   (f)
$$g(t) = e^{-3t}\, f(t)$$

   (g)
$$g(t) = [f \circledast f](t).$$

3. Determine, for each of the following cases of $F(s)$, the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F\}$.

   (a)
$$F(s) = \frac{s+2}{s^2+7s+12}$$

(b)

$$F(s) = \frac{s+1}{s^3 + 5s^2 + 7s}$$

(c)

$$F(s) = \frac{3s^2 + 2s + 1}{s^3 + 5s^2 + 8s + 4}$$

(d)

$$F(s) = \frac{s^2 + 1}{s^2 + 7s + 12}$$

(e)

$$F(s) = \frac{s + e^{-s}}{s^2 + 3s + 2}$$

4. The relationship between two signals $y(t)$ and $u(t)$ is known to be governed by the following differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 7\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 10y(t) = 5\frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t)$$

   (a) Compute the transfer function $H(s)$ relating $Y(s) = \mathcal{L}\{y\}$ to $U(s) = \mathcal{L}\{u\}$.
   (b) Compute the impulse response $h(t)$ governing the relationship $y(t) = [h \circledast u](t)$ between $y(t)$ and $u(t)$.

5. The response $y(t)$ of a system $\mathcal{S}$ to a signal $u(t)$ is given by the following differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 4\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 3y(t) = u(t).$$

   Compute the response $y(t)$ for $t \geq 0$ for the following cases ($\mathbf{1}(t)$ is the unit step)

   (a)

$$y(0) = -2, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right|_{t=0} = 1, \qquad u(t) = 0.$$

   (b)

$$y(0) = 0, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right|_{t=0} = 0, \qquad u(t) = \delta(t).$$

   (c)

$$y(0) = 0, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right|_{t=0} = 0, \qquad u(t) = \mathbf{1}(t).$$

   (d)

$$y(0) = -2, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right|_{t=0} = 1, \qquad u(t) = \mathbf{1}(t).$$