ELEC240 - Tutorial # 6

1. Define $r(t)$ to be the rectangular function

$$r(t) \triangleq \begin{cases} \frac{1}{2\tau} & ; |t| \leq \tau \\ 0 & ; |t| > \tau \end{cases}$$

Sketch $r(t)$, derive it's Fourier Transform $R(\omega)$, and sketch that as well.

2. Define $\triangle\ (t)$ to be the triangular function

$$\triangle\ (t) = \begin{cases} 1 - \frac{|t|}{2\tau} & ; |t| \leq 2\tau \\ 0 & ; |t| > 2\tau \end{cases}$$

Sketch $\triangle(t)$, derive it's Fourier Transform $\triangle(\omega)$, and sketch that as well.

3. Sketch the convolution

$$[r \circledast r](t)$$

How are $r(t)$ and $\triangle(t)$ related ? From this, how should the Fourier transforms $R(\omega)$ and $\triangle(\omega)$ be related ?

4. Consider the square signal $r(t)$ defined in question 1 of and the triangular signal $\triangle(t)$ defined in question 2. Derive an expression for, and plot the signal $x(t)$ given by

$$x(t) = [r \circledast \triangle]\,(t)$$

Derive an expression for the Fourier Transform $X(\omega)$.

5. Consider the signal's $g(t)$ and $f(t)$ plotted in figure 1. Derive their Fourier transforms $G(\omega)$ and $F(\omega)$ and plot their amplitude response with respect to frequency ω.

6. Consider the signal

$$y(t) = \begin{cases} |t| & ; |t| \leq 1 \\ 2 - |t| & ; |t| \in (1, 2] \\ 0 & ; \text{Otherwise} \end{cases}$$

Sketch $y(t)$. Derive its Fourier Transform $Y(\omega)$ and sketch that as well. Try to minimise the amount of calculation by using known results.

7. (a) Consider the signal $f(t)$ shown on the left in figure 2. Derive it's Fourier transform $F(\omega)$ and sketch it's magnitude $|F(\omega)|$.
   (b) Consider the signal $x(t)$ shown on the right in figure 2. Derive it's Exponential Fourier series representation.
   (c) Use the answer to the preceding question to derive the Fourier transform $X(\omega)$ of the signal $x(t)$ shown on the right in figure 2. Plot it's magnitude $|X(\omega)|$.

8. Put

$$\begin{aligned} f(t) &= \sum_{k=-\infty}^{\infty} g(t + 2k) \\ g(t) &= \begin{cases} e^{-2|t|} & ; |t| \leq 1 \\ 0 & ; \text{otherwise} \end{cases} \end{aligned}$$

Calculate the Fourier series representation for $f(t)$.

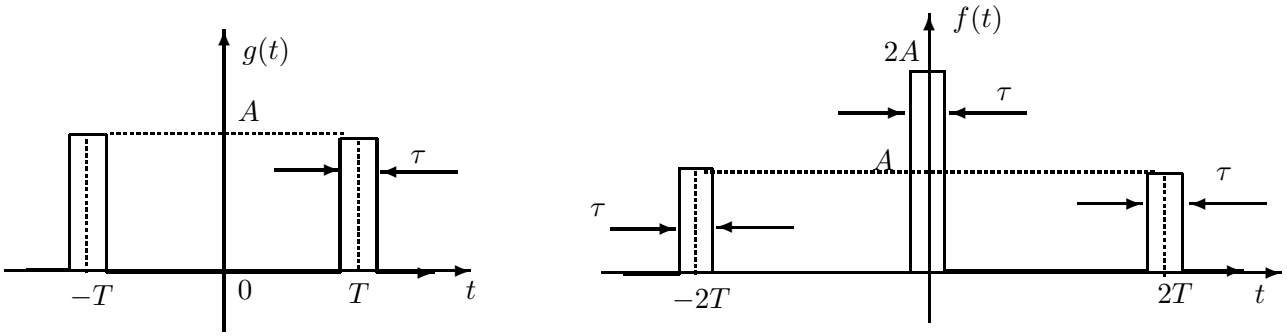

Figure 1: *Signals* $g(t)$ *and* $f(t)$

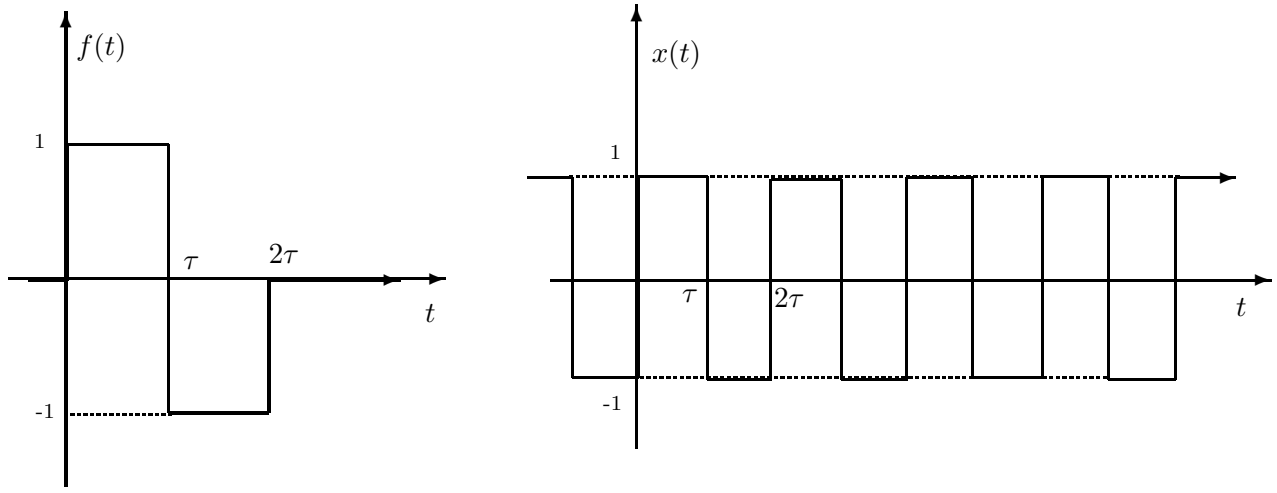

Figure 2: *Signals* $f(t)$ *and* $x(t)$.

9. A full wave rectifier converts a 5Hz sinusoidal signal into the signal

$$y(t) = |2\sin 10\pi t|\,.$$

Sketch this signal. Calculate its Fourier Transform $Y(\omega)$ and sketch it as well. What fraction of the power in the signal $y(t)$ is in the spectral region below 50Hz?