ELEC2400, Assignment Number 1

1. **(25 marks Total)**

   Compute and sketch the convolution

   $$y(t) = [h \circledast u](t) = \int_{-\infty}^{\infty} h(t-\sigma)u(\sigma)\,\mathrm{d}\sigma$$

   for the following cases

   (a) **(5 marks)**

   $$\begin{aligned} h(t) &= e^{-|t|} \\ u(t) &= \delta(t-T), \qquad T > 0 \end{aligned}$$

   (b) **(10 marks)**

   $$\begin{aligned} h(t) &= \mathbf{1}(t) \cdot e^{-2t} \\ u(t) &= \begin{cases} 1 & ; t \in [0,2] \\ -1 & ; t \in [2,4] \\ 0 & ; \text{Otherwise} \end{cases} \end{aligned}$$

   (c) **(10 marks)**

   $$\begin{aligned} h(t) &= \mathbf{1}(t) \cdot e^{-3t} \\ u(t) &= \begin{cases} 1 & ; t \in [0,1] \\ -1 & ; t \in [2,3] \\ 0 & ; \text{Otherwise} \end{cases} \end{aligned}$$

2. **(25 marks total)**

   A system has impulse response

   $$h(t) = \begin{cases} 2e^{-3t} + e^{-t} & ; t \geq 0 \\ 0 & ; \text{Otherwise} \end{cases}$$

   and is fed the input signal

   $$u(t) = \begin{cases} 3\cos 4\pi f t & ; t \geq 0 \\ 0 & ; \text{Otherwise} \end{cases}$$

(a) (**3 marks**) Is this a causal or a non-causal linear system? Explain your answer in terms a convolution relationship.

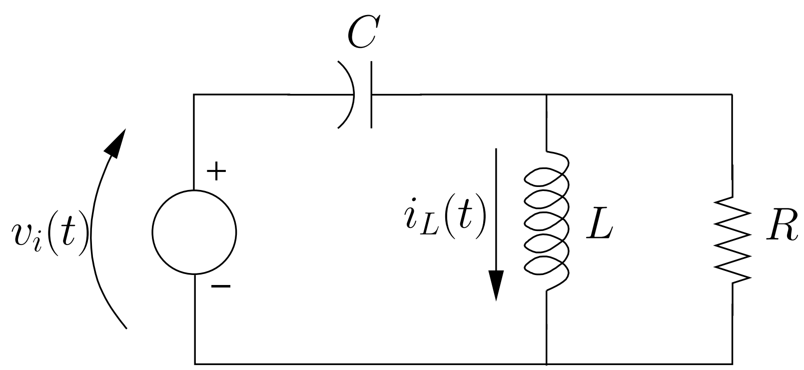
(b) (**3 marks**) What is the transfer function $H(s)$ for this system?

(c) (**4 marks**) In steady state (in the limit as $t \to \infty$) what is the output $y(t) = [h \circledast u](t)$ of the system?

(d) (**10 marks**) Work out the complete (i.e. not just steady state, but also including transient component) response $y(t) = [h \circledast u](t)$ using any method you wish.

(e) (**5 marks**) Write down a differential equation relating the output $y(t)$ to the input $u(t)$ for arbitrary $u(t)$.

3. (**35 marks total**) Consider the circuit shown below



(a) (**7 marks**) Derive a differential equation relationship between the voltage source signal $u(t) = v_i(t)$ and the signal $y(t) = i_L(t)$ which is the current flowing through the inductor.

(b) (**5 marks**) Present an observer canonical form state-space description of this same differential equation relationship.

(c) (**5 marks**) For the particular case of

$$C = 2083.333\mu\text{F}, \qquad L = 32\text{H}, \qquad R = 60\Omega$$

provide an expression for the matrix exponential $e^{\mathbf{A}t}$ associated with the preceding state space realisation.

(d) (**8 marks**) Use the results of the preceding two parts to derive the system impulse response $h(t)$.

(e) (**10 marks**) Again using the results of preceding parts, use state-space methods to calculate and plot a the signal $i_L(t)$ that, at time $t = 0$ satisfies

$$i_L(0) = 1\,\text{V}, \qquad \left.\frac{\mathrm{d}}{\mathrm{d}t} i_L(t)\right|_{t=0} = -2\,\text{V/s}$$

when $v(t)$ is the signal

$$v(t) = \begin{cases} 1 & ; |t - 1.5| \le 0.5 \\ 0 & ; \text{Otherwise} \end{cases}$$

4. (**15 marks total**)

The Bode magnitude curve of a system with transfer function $G(s)$ can be approximated by straight-line asymptotes, as shown in Figure 1. No further changes in the gradients of the asymptotes occurs for frequencies outside the range shown.

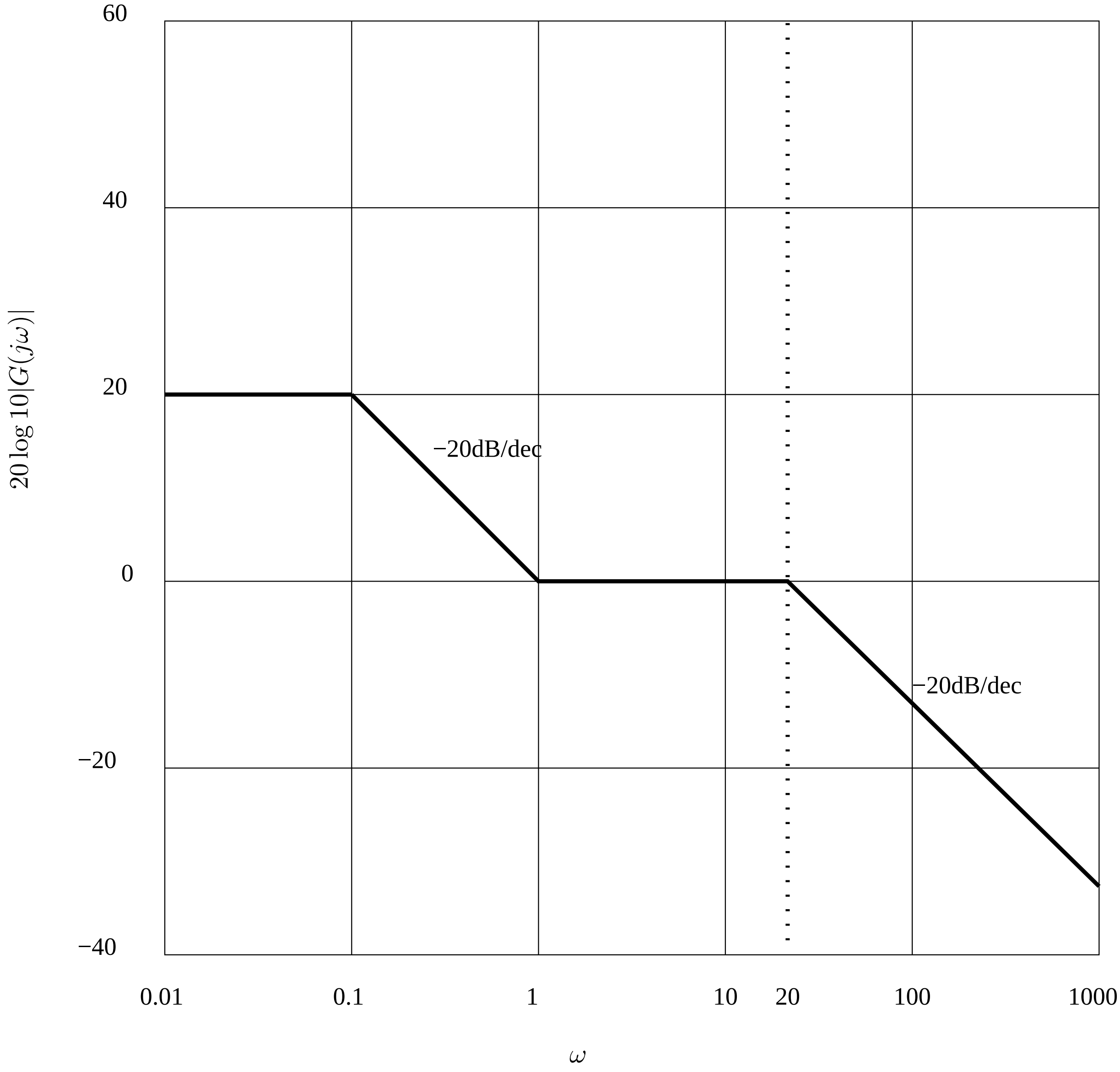


Figure 1: *Straight line asymptotic approximation to Bode magnitude curve.*

(a) (**3 marks**) What is a possible transfer function for this system?

(b) (**5 marks**) Calculate a state-space representation for this system.

(c) (**7 marks**) Sketch the Bode phase curve for this system.