# EXIT Charts: The Very Basics

Sarah Johnson

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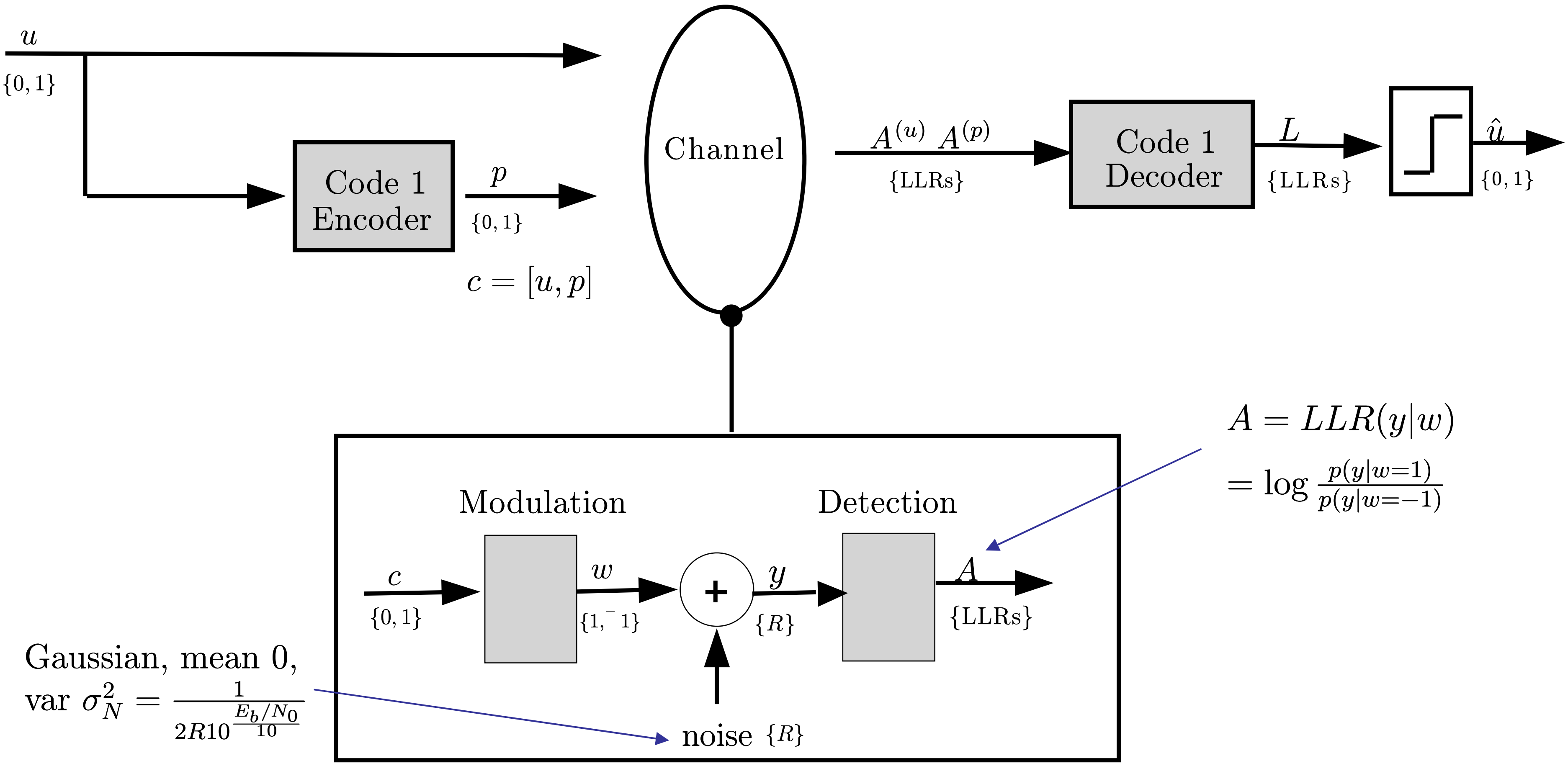
# EXIT Charts: The Very Basics

- Convolutional codes and Turbo codes
- Mutual information transfer curves
- EXIT charts
- The benefits of EXIT charts

# Convolutional Coding

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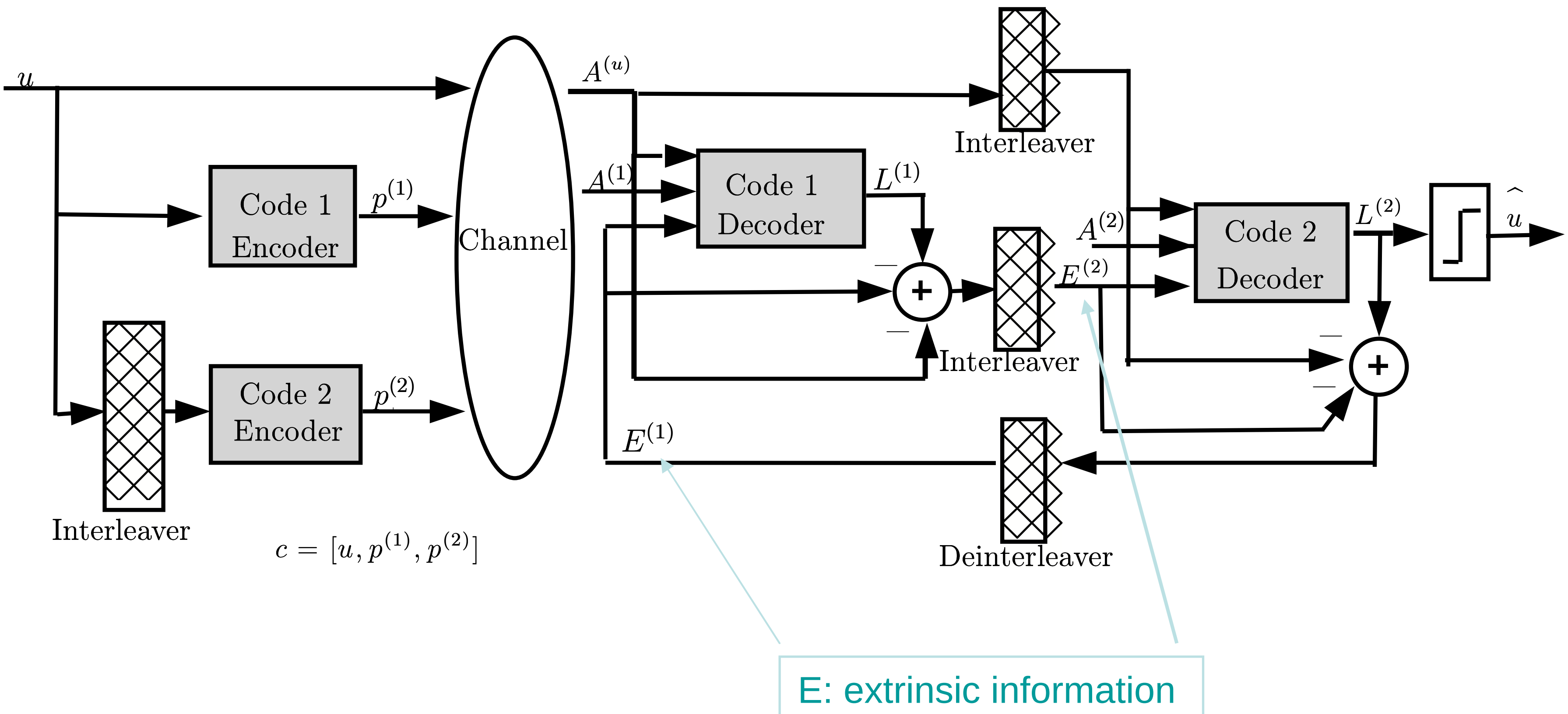
# Turbo Coding





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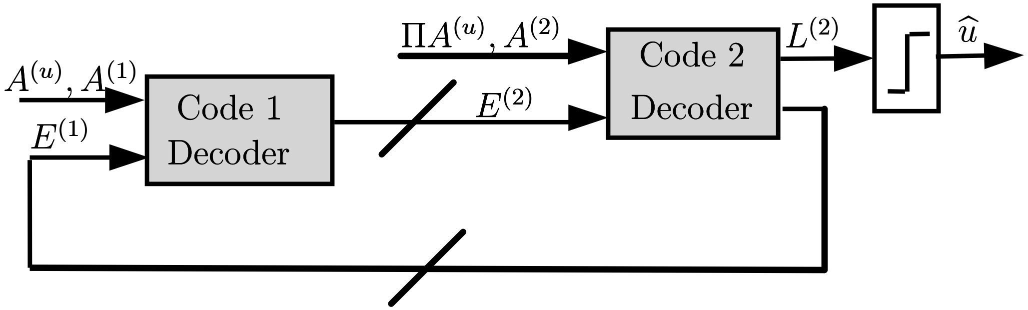
# Mutual information transfer

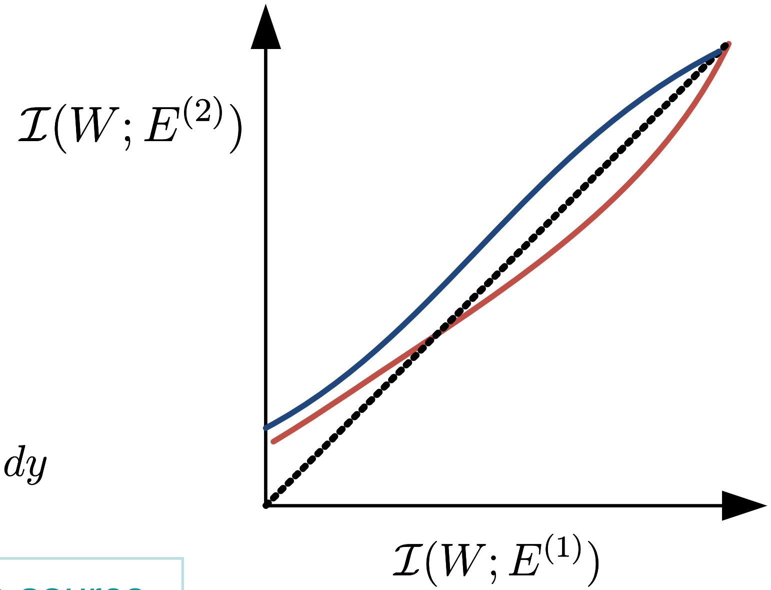



- We want to quantify the ‘improvement’ over each iteration
- Do this by measuring the mutual information between E and W

$$\mathcal{I}(X;Y) = \int\int p(x,y)\log_2 \frac{p(x,y)}{p(x)p(y)} dxdy$$

$$\mathcal{I}(W;E) = \int_{y\in E} \sum_{w\in\pm 1} p(w,y)\log_2 \frac{p(w,y)}{p(w)p(y)} dy$$

$$\mathcal{I}(W;E) = \frac{1}{2}\int_{y\in E} \sum_{w\in\pm 1} p(y|w)\log_2 \frac{p(y|w)}{\frac{1}{2}(p(y|1)+p(y|-1))} dy$$




W: binary equiprobable source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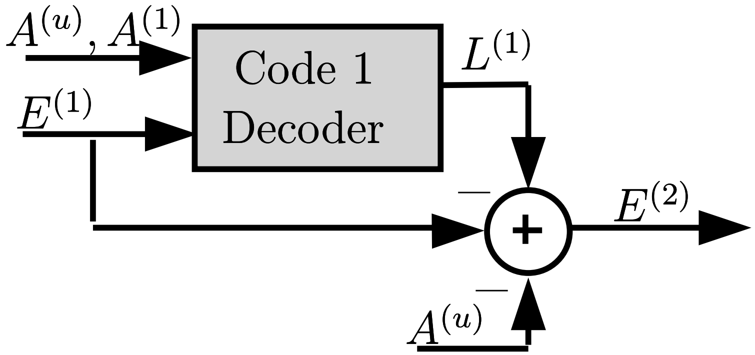
# Mutual information transfer




Gaussian, mean 0,
var $\sigma_N^2 = \frac{1}{2R10^{\frac{E_b/N_0}{10}}}$

$$
\begin{aligned}
A &= \ln \frac{p(w=1|y)}{p(w=-1|y)} \\
&= \ln \frac{p(w=1)p(y|w=1)/p(y)}{p(w=^{-}1)p(y|w=^{-}1)/p(y)} \\
&= \ln \frac{e^{\frac{-1}{2\sigma^2}(y-1)^2}}{e^{\frac{-1}{2\sigma^2}(y+1)^2}} = \frac{2y}{\sigma^2}
\end{aligned}
$$

Fix the channel noise and generate $A^{(u)}$ and $A^{(1)}$: $\mathbf{y} = \mathbf{w} + \mathbf{n_A}$, $A = \frac{2}{\sigma_N^2}\mathbf{y}$

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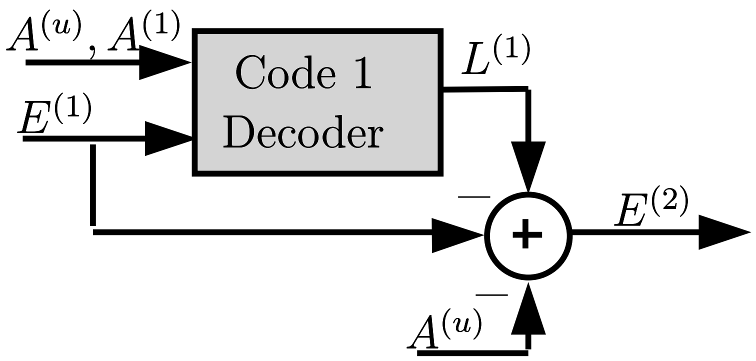
# Mutual information transfer




$$\mathcal{I}(W;E) = \frac{1}{2}\int_{y\in E^{(1)}} \sum_{w\in\pm 1} p(y|w)\log_2 \frac{p(y|w)}{\frac{1}{2}\left[p(y|1)+p(y|-1)\right]}dy$$

$$p(y|w) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{1}{2\sigma^2}(y-\mu w)^2}$$

Fix the channel noise and generate $A^{(u)}$ and $A^{(1)}$: $\mathbf{y} = \mathbf{w} + \mathbf{n_A}, \quad A = \frac{2}{\sigma_N^2}\mathbf{y}$

For a few different values of $\mathcal{I}\{E^{(1)}\}$:

1. Generate $E^{(1)}$ with a gaussian pdf: $E^{(1)} = \mu_E \mathbf{w} + n_E$

Gaussian, mean 0, var ${\sigma_E}^2 = 2\mu_E$

then $\mathcal{I}(W;E^{(1)}) = 1 - \int_{y\in E^{(1)}} \frac{e^{\frac{-1}{2{\sigma_E}^2}(y-\mu_E)^2}}{\sqrt{2\pi\sigma_E}} \log_2(1-e^{-y})dy$

Independent noisy observations of W

2. Run the BCJR decoder. Since $w = +1$ a histogram of $E^{(2)}$ values provides $p(E^{(2)}|w=1)$, and since $p(E^{(2)}|w=-1) = p(-E^{(2)}|w=1)$ we can calculate $\mathcal{I}(W:E^{(2)})$ numerically

Plot $\mathcal{I}(W;E^{(2)})$ Vs $\mathcal{I}(W;E^{(1)})$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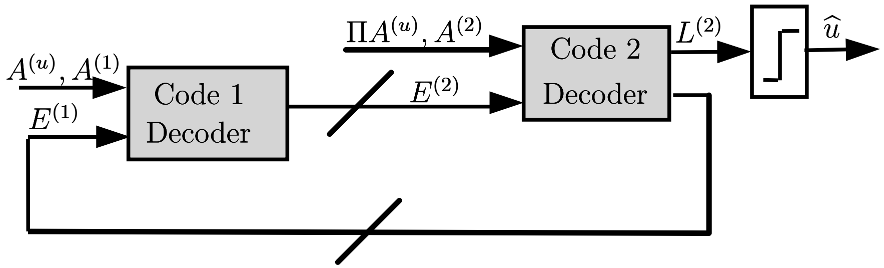
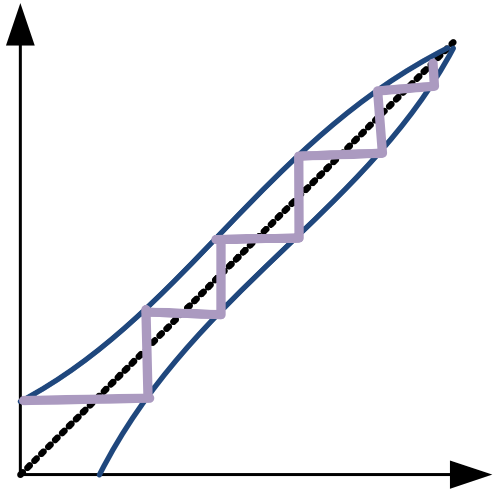
# EXIT Charts




Plot $\mathcal{I}(W; E^{(2)})$ Vs $\mathcal{I}(W; E^{(1)})$ and $\mathcal{I}(W; E^{(1)})$ Vs $\mathcal{I}(W; E^{(2)})$



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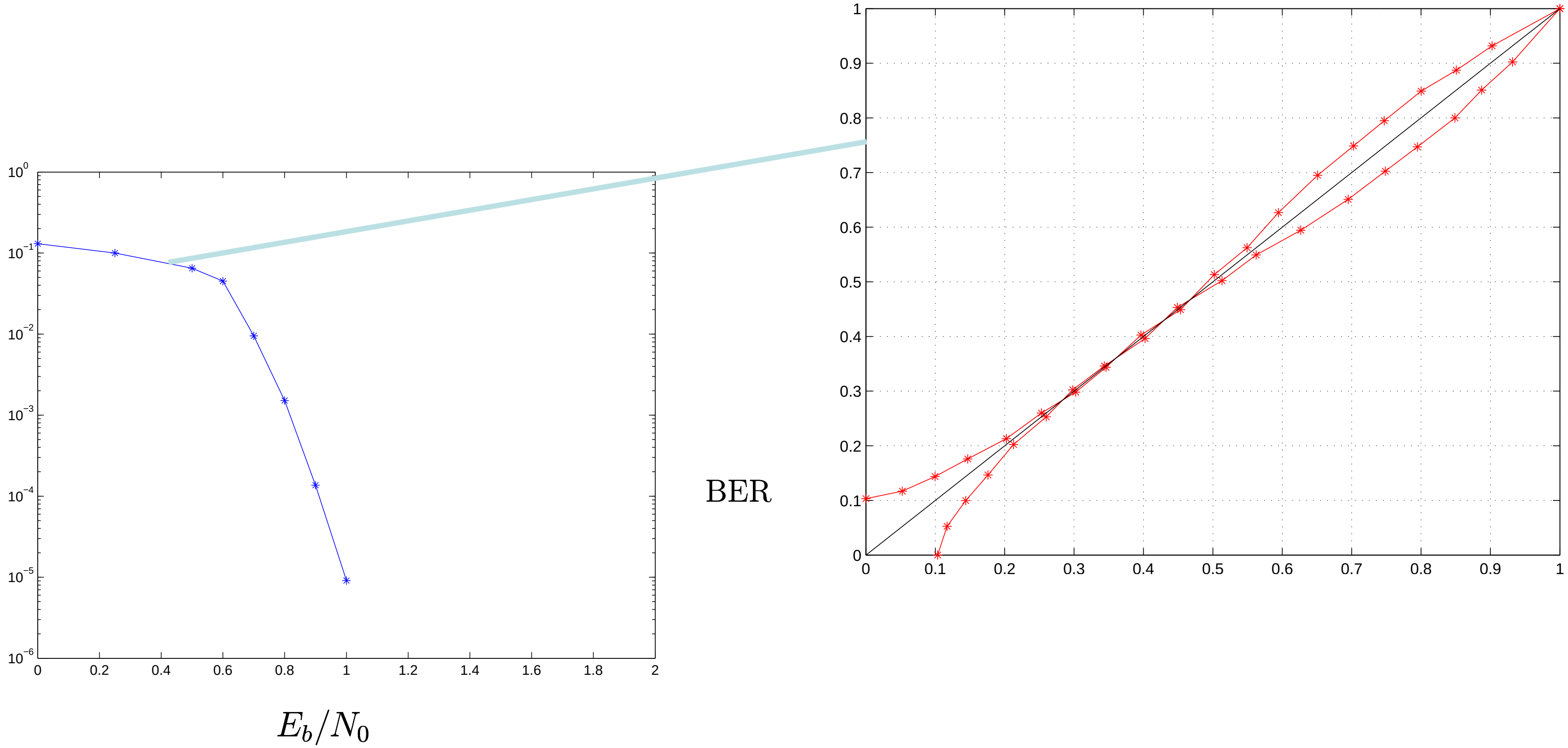


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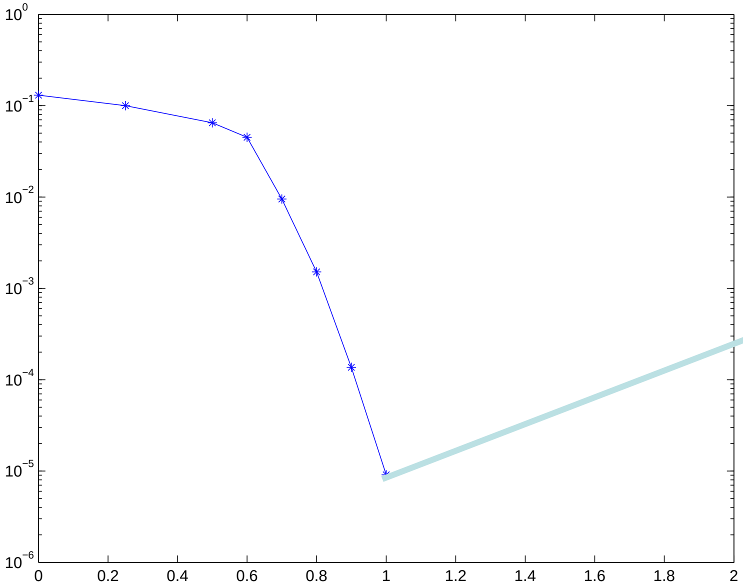
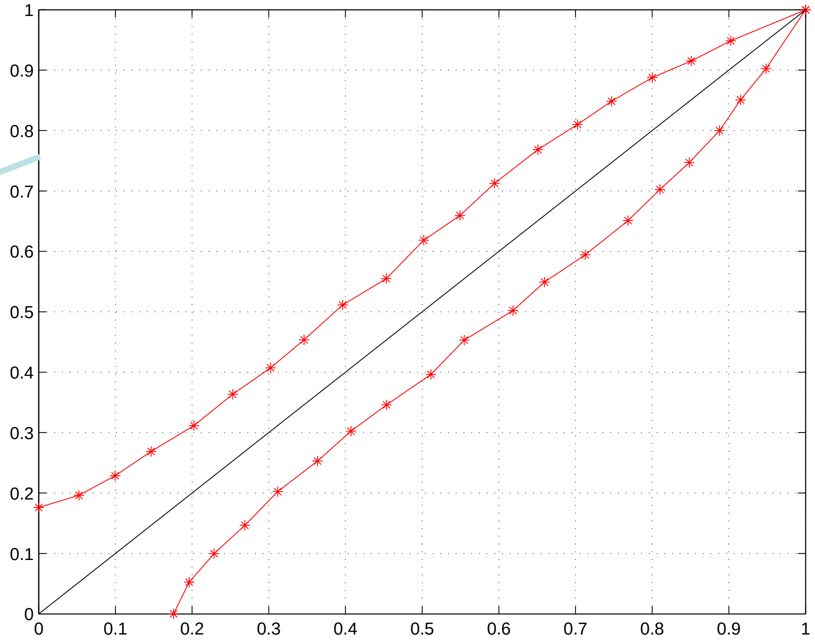



$E_b/N_0$

BER

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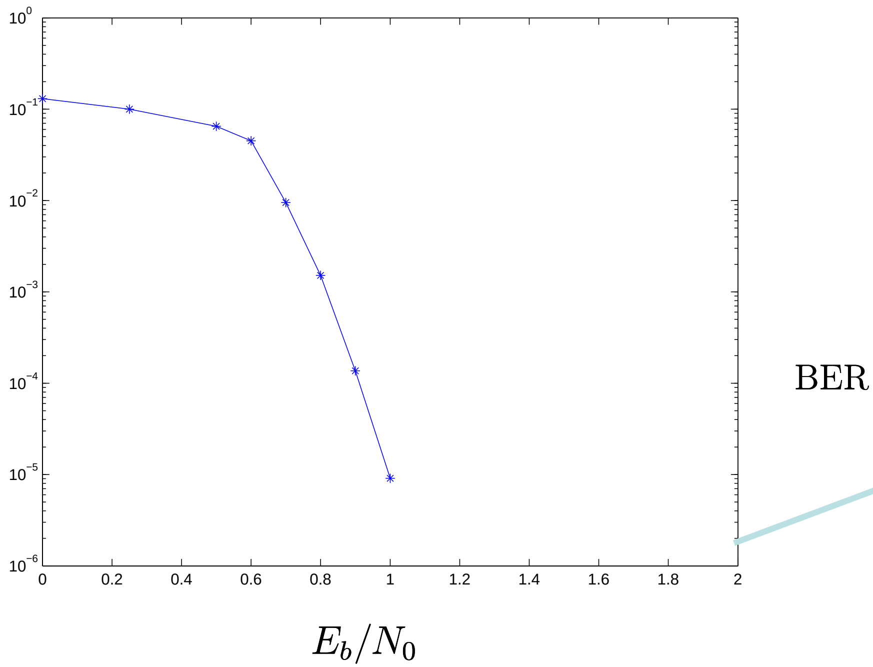


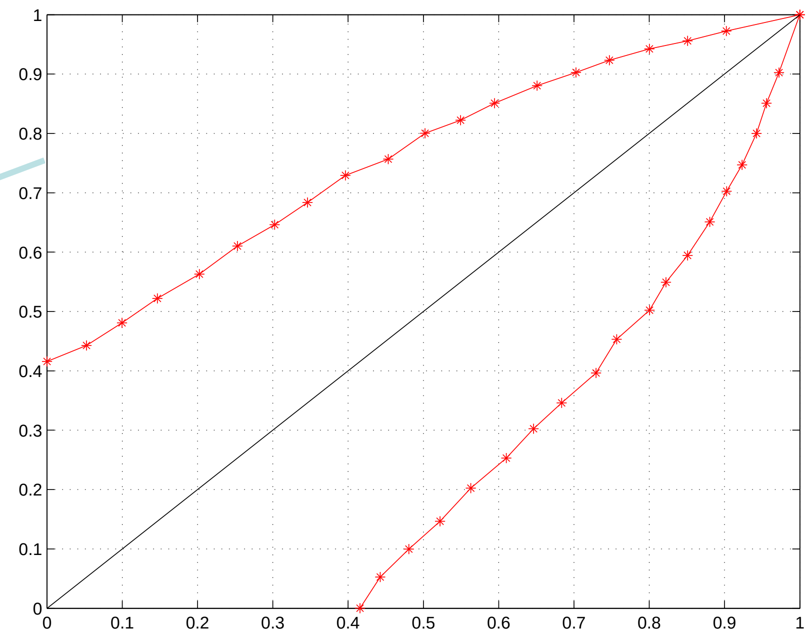


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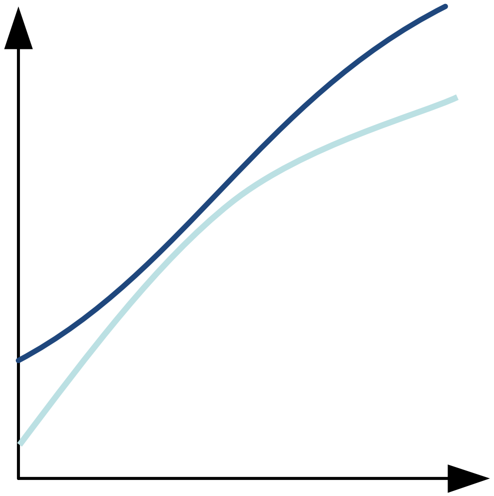
# EXIT Charts

- Understand the performance of iterative ‘turbo’ schemes
- Predict the decoder’s ‘threshold’ without having to simulate the whole decoder
- As a tool for designing the component codes



- Ignores the impact of the interleaver – assumes it is ideal