Reported Thresholds and BER Performance for LDPC and LDPC-Like Codes

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Table 1 gives the reported performance of known low-density parity-check (LDPC) and related codes. Shown is their theoretical threshold and, where known, actual bit error rate (BER) performances.

The general trend we can see in the table is: for high rates regular LDPC codes can provide thresholds reasonably close to capacity while for medium rates irregular LDPC codes are required if you capacity-approaching thresholds. Meanwhile for low rates even irregular LDPC codes do not have great thresholds and more complex codes are required.

The selection of codes at rate-1/2 gives a good example of how threshold varies with degree distribution and actual BER performance varies with code length. To achieve an actual BER performance of 1 bit error in 10^6 transmitted bits at an E_b/N_0 which is anywhere near the theoretical threshold requires codes of length at least 10^5 bits. For performances a very small fraction of a decibel from capacity (i.e. 0.04dB) very high weight bit nodes (100 edges for some nodes) and codeword lengths of 10^7 bits are required.

Table 1: Theoretical and reported performance of low rate codes on the BI-AWGN channel. All noise levels given are E_b/N_0 in dB

Rate	Shannon Limit (E_b/N_0)	Code Type	Threshold (E_b/N_0)	Simulated Performance (E_b/N_0 at which a BER of 10^{-6} is achieved)
0	-1.6			
0.003	-1.57	Hadamard-LDPC		-1.44 (Π = 650,000) [1]
0.008	-1.55	Hadamard-LDPC		-1.38 (Π = 238,000) [1]
0.0107	-1.54	Systematic ZH	-1.25	-1.15 (Π = 65,536) [2]
0.034	-1.48	Systematic ZH	-1.1	-0.98 (Π = 65,536) [2]
1/20	-1.42	Hadamard-LDPC		-1.18 (Π = 65,536) [1]
1/10	-1.285	Multi-edge LDPC	-1.09	
		AR4A	-1.028	[3]
		LDPC ($d_l = 10$)	-0.9950	
0.108	-1.25	Systematic ZH	-0.6	-0.44 (Π = 65,536) [2]
1/8	-1.206	AR3A	-0.98	[3]
1/6	-1.073	AR3A	-0.818	0.6 (N = 6,144) [3]
1/5	-0.963	AR3A	-0.804	[3]
		LDPC ($d_l = 10$)	-0.6930	
1/4	-0.794	AR3A	-0.604	0.75 (N = 4,096)[3]
		LDPC (3,4 reg.)	1.1060	
		LDPC ($d_l = 10$)	-0.544	
1/3	-0.495	AR3A	-0.13	1.15 (N = 3,072)[3]
		LDPC (4,6 reg.)	0.9050	
		LDPC (McKay)		1 ($N = 15,000$) [4]

Rate	Shannon Limit (E_b/N_0)	Code Type	Threshold (E_b/N_0)	Simulated Performance (E_b/N_0 at which a BER of 10^{-6} is achieved)
1/2	0.187	LDPC (3,6 reg.)	1.1	1.6 ($N = 10^4$) [5]
		LDPC (3,6 reg.)	1.1	1.16 ($N = 10^6$)
		RA (4,4 reg.)		1.4 ($N = 10^4$) [5]
		precoded (3,6 reg.)	0.87	1.4 ($N = 8,192$) [6]
		AR4A	0.56	1.2 ($N = 8,192$) [3]
		Multi-edge LDPC	0.355	1.08 ($N =$ 10,240)
		LDPC ($d_l = 10$)	0.39	
		LDPC ($d_l = 100$)	0.22	0.24 ($N = 10^7$) [7]
		IRA ($d_l = 100$)	0.246	0.4 ($N = 10^5$) [5]
		LDPC ($d_l = 200$)	0.2018	0.23 ($N = 10^7$) [7]
		LDPC ($d_l = 8000$)	0.1916	
2/3	1.059	LDPC (3,9 reg.)	1.752	
		AR4A	1.414	2.1 ($N = 6,144$) [3]
3/4	1.626	LDPC (3,12 reg.)	2.229	
		precoded (3,12 reg.)	2.02	[6]
		AR4A	1.980	2.6 ($N = 5,440$) [3]
		EG LDPC (4,16 reg.)		2.8 ($N = 8,176$) [8]
4/5	2.040	LDPC (3,15 reg.)	2.594	
		AR4A	2.396	3 ($N = 5,440$) [3]
0.812	2.3	extnd. EG LDPC (4,21/22 almost reg.)		2.9 ($N = 65,520$) [8]
5/6	2.362	LDPC (3,18 reg.)	2.882	
		AR4A	2.717	3.3 ($N = 4,896$) [3]
6/7	2.625	LDPC (3,21 reg.)	3.117	
0.875	2.8	LDPC (4,33 reg.)		3.8 ($N = 10,240$)
8/9	3.003	AR4A	3.386	[3]
0.9	3.198	LDPC (3,30 reg.)	3.642	
0.969	4.85	extnd. EG LDPC (4,128 regular)		5.19 ($N = 52,4256$) [8]

Notes for Table 1:

1. The "Shannon limit" gives the largest noise level that can be corrected by any possible forward error correction system at that code rate. Taking the limit as rate goes to zero gives the fundamental Shannon bound which says no communication system, even if it has infinite bandwidth, can operate below an Eb/No of -1.6 dB.

2. The "Threshold" is the theoretical performance for an ensemble of infinite length codes with unlimited decoder iterations. An "ensemble" of LDPC codes is the set of all possible codes with certain parameters (usually the number of non-zero entries in each row and column of the parity-check matrix H). The threshold is found using density evolution or EXIT chart analysis. All thresholds in the table are reported for full soft-decision sum-product (belief propogation) decoding.

3. For the Systematic ZH and Hadamard-LDPC codes the simulated E_b/N_0

values are reported for a BER of 10^{-5} since results were not reported for BERs below this value. If these codes do perform at 10^{-6} (I.e., if the error floor does not kick in above this level) the E_b/N_0 values will be slightly larger.

4. Regular LDPC codes are specified by (j, k reg.) and are defined as having all columns of the parity-check matrix H containing j non-zero entries and all rows of H having k non-zero entries.

5. Irregular LDPC codes have variable row and column weights and were introduced to drastically improve the thresholds of LDPC ensembles. The maximum number of non-zero entries in any one column (d_l) is specified as an indication of their complexity. (As a guide, the larger d_l the more complex the implementation and the better the threshold).

6. Many of the LDPC thresholds are from [7]. Others have been calculated using my own density evolution implementation.

7. The repeat-accumulate (RA) and irregular repeat-accumulate codes (IRA) are specified similarly to the LDPC codes.

8. The AR3A and AR4A codes are almost-regular LDPC codes specified by RA-like protographs (small graphs which are repeated many times to form the overall code).

9. The rate-1/10 multi-edge LDPC code is from: Richardson and Urbanke, "Multi-Edge Type LDPC Codes" Unpublished.

References

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