Comment on Paper by Reinelt and Ljung

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This paper is an interesting and novel contribution to the important area of marrying robust control design to model formats provided by available system identification methods. This latter problem has generated significant research interest over the last decade, and is still of high interest as this paper, and other recent contributions such as [1, 2] and their extensive bibliographies attest.

A fundamental and recurring theme in this area, is that of the bias/variance tradeoff that is commonly assumed inevitable, and which necessitates the consideration of error bounds accounting for the concomitant undermodelled dynamics. These are labelled Δ in the paper under discussion.

However, there is another perspective which I believe is intriguing. Suppose that consideration of undermodelled dynamics is avoided by full order modelling. As mentioned above, a central tenet of the area in which the paper under discussion sits, is that this will imply an intolerably large variance error, and in fact one would be better to fit a low order model to minimise the variance, while then having to account for a bias error. This is done on the grounds of it being expected to minimise the total error to one smaller than that of the variance error associated with a full order model.

But is this true? For example, a fundamental aspect of system identification is that *optimal* accuracy is achieved via a maximum likelihood approach, and this demands *full* order modelling. Furthermore, another advantage of this approach is that error bounds are then delivered via standard confidence regions.

If a low order model is then required, it may be obtained by subsequent model reduction. Furthermore, error bounds on this reduced order model are simply computed from those of the original high order modelling via multiplication by the sensitivity of the model reduction mapping.

As shown in [7, 6] for certain model structures, this procedure results in superior accuracy than one of estimating the low order model directly. Furthermore, as discussed in [4] this principle is likely to be far more general since if the reduced order model is a smooth function of the full order model (it will be, for example, under an L_2 criterion), then this reduced order model is *also* a maximum likelihood estimate, and hence also of optimal accuracy.

For instance, consider the simulation example of the paper under review, which originally derives from [3]. Via a Maximum Likelihood perspective, an Output-Error model structure should be estimated, which is shown as the solid line in figure 1. Note from the Bode plot that, compared to figure 2 of the paper under discussion, the estimation error associated with this Maximum Likelihood approach is appreciably less than that arising from bias error associated with undermodelling.

Furthermore, note from the Nyquist plot that the problem of control design for this system seems rather benign, and indeed the simple PI controller that is easily designable from the information shown in figure 1 of

$$u_t = \left[2 + \frac{0.1}{1 - q^{-1}}\right](y_t - r_t) \tag{1}$$

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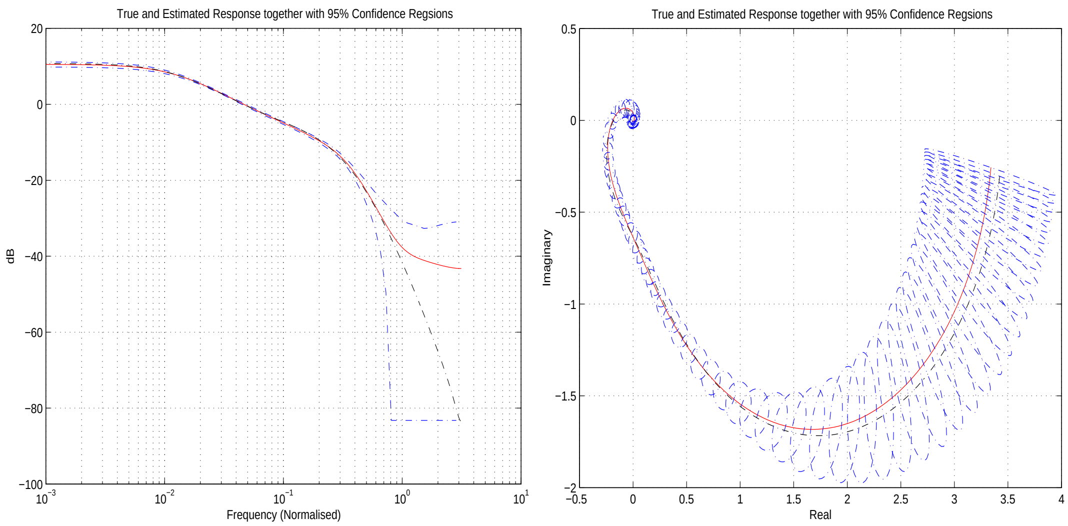


(a) True and Estimated Bode

(b) True and Estimated Nyquist

Figure 1: *True and Estimated Responses. Solid line is estimate, dashed in true response, dash dot lines/ellipses are 95% confidence regions.*

seems to perform to the robustness specification shown in figure 4 of the paper under discussion, and with step response as shown in 2(a) This discussion is not meant to suggest any flaw in the problem formulation or approach used in the paper by Reinelt and Ljung. Figures 1,2(a) pertain to only one example, and general yet computationally feasible design frameworks such as those proposed in this work are greatly needed. However, the above observations do re-enforce the importance of the call in [4] for benchmark problems to aid in the profiling of different estimation/control strategies.

To conclude this comment, it could be argued that one of the most fundamental questions that could be asked in the context of the interplay between identification and control design, might be along the lines of:

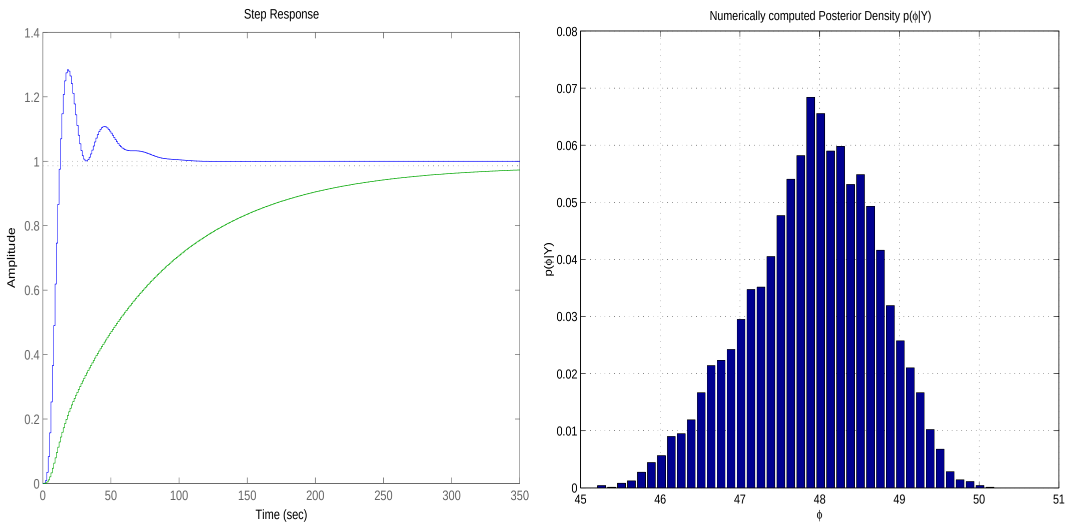
> Given the prior assumption of the underlying system $G(q)$ being linear and of a certain order, and given that I propose to use this given controller $K(q)$, then what information is in the observed behaviour of the system that will allow me judge the performance of my proposed controller?

The approach of the work by Reinelt and Ljung to this question is to use an underlying estimated model together with error bounds to formulate error bounds on, for example, sensitivity functions.

However, another very fundamental approach would be to compute the posterior density of the performance criterion of interest, conditional on the data and prior assumptions of system linearity. That is, if the phase margin $\phi(K)$ for a given controller were required, then one could seek to find the posterior density

$$p(\phi(K) \mid Y). \tag{2}$$

where Y denotes observed data together with any prior assumptions, such as underlying model order, linearity and so on. From a Bayesian perspective, consideration of the posterior density (2) is optimal, since it encapsulates everything that can be said about the information content in the data relative to the achieved phase margin.



(a) Step Response

(b) Posterior $p(\phi|Y)$

Figure 2: *Figure (a): Achieved Step response on true system - faster response is closed loop response, slower response is open loop response. Figure (b): Posterior density $p(\phi(K)|Y)$ of achieved phase margin given observed system response*

However, to date the examination of quantities such as (2) has not been pursued on the grounds of it being too difficult to compute them. Certainly, any analytical attempt to translate error bounds on G to phase margin seems impossible due to the non-differentiability of the mapping.

In relation to this, and the area in which the work of Rienelt and Ljung has contributed, it is perhaps interesting to note the new so-called Markov-Chain Monte–Carlo (MCMC) methods provide a means to compute quantities such as (2).

For example, figure 2(b) shows this density for the simulation example and PI controller discussed earlier. Clearly, this indicates that the observed system behaviour strongly supports a hypothesis that the PI controller (1) will, when implemented, achieve a phase margin specification of $\phi(K) > 45°$.

The methods used to compute this density are simple, require no new data to be collected, but are computationally intensive and probably unsuitable for on-line applications. They are presented in [5], and in the authors opinion provide one new option for future work in this area of linking identification and control.

Acknowledgement The responsibility for these comments rests with the author, but he would like to acknowledge that they are influenced by numerous discussions with, and useful insights provided by Håkan Hjalmarsson.

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