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Brief Paper

Anti-Windup Designs for Multivariable Controllers*

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Key Words—Control nonlinearities; multivariable control; saturation; anti-windup; direction preserving.

Abstract—This paper addresses two important aspects of anti-windup (AW) designs, namely the parametrization of linear AW compensators, and the role of artificial nonlinearity (AN) in the design of AW compensators for multivariable systems. For the first issue, a simple parametrization is given using the classical feedback structure in the framework of constrained unity-feedback multivariable control systems. For the second issue, two existing AN designs for coordinating plant inputs whenever one plant input enters saturation are reviewed. A comparative simulation study illustrates that the conditioning technique, enhanced by optimal AN design, gives the best tracking performance among different existing methods. © 1998 Elsevier Science Ltd. All rights reserved.

1. *Introduction*

The control inputs for all industrial processes are subject to hard physical constraints. For instance, controllers may be constrained to produce outputs in the range 0–10 V or 0–20 mA, valves cannot be opened more than 100% or less than 0%, motor-driven actuators have limited speeds, and so on. Such constraints are usually referred to as plant input limitations. Likewise, it is common practice to switch from manual to automatic mode, or between different controllers. Such mode switches are usually referred to as plant input substitutions.

As a result of limitations and substitutions, the real plant input may differ from the controller output. When this happens, the closed-loop performance may be significantly degraded in comparison with the expected performance of the controller designed to operate in a linear regime. This performance deterioration is referred to as windup. In the case of substitution, the difference between the outputs of different controllers may result in large jumps in the plant input, resulting in poor tracking performance. Mode switching leading to such phenomena is referred to as "bump transfer".

One way of handling the problem of windup is to incorporate input limitations into the initial control design process. This approach can be quite involved, however, and the resulting (nonlinear) control laws can be very complicated. Moreover, the nonlinearities of the actuator are not always known *a priori*. A more common approach in practice is to perform a linear control design, then to add extra feedback compensation at the stage of control implementation. As this form of compensation aims to reduce the undesirable effects of windup, it is referred to as anti-windup (AW).

In the case of mode switching, methods aiming to minimize the jump at the plant input are referred to as bumpless transfer (BLT) methods. However, minimizing the jump in the manipulated variables is not always the best course of action; what is of primary importance is the resulting closed-loop tracking performance. Thus we refer to methods which not only reduce the jump at the plant input but also maintain acceptable tracking performance as conditioned transfer (CT) methods. AW strategies are usually treated as bumpless transfer methods, and indeed AW methods will usually reduce the jump at the plant input during mode switching. However, it should be pointed out that AW does not necessarily imply bumpless transfer. In fact, if AW aims at improving tracking performance during actuator saturation, then it will imply conditioned transfer in the case of mode switching.

The topic of AW design has been studied for some four decades by many authors. The most popular techniques are described in Hanus (1989) and Kothare *et al.* (1994), and the references therein. In Kothare *et al.* (1994), a general framework for AW design is presented, in which all known AW compensation schemes arise as special cases.

Recently, we have provided guidelines for designing AW, BLT and CT compensation for PID controllers (Peng *et al.*, 1996). Subsequent investigation has revealed that the conclusions can be extended to more general controllers. As a result, a key objective in this paper is to provide a simple parameterization for AW designs. Since the proposed parameterization is based on the classical feedback control scheme, it has some advantages over the framework proposed by Kothare *et al.* (1994). First, the AW design can be explained in a natural and straightforward manner. Second, different existing AW designs can easily be compared, and their advantages and disadvantages, readily understood.

One problem associated with saturation in multivariable (multi-input, multi-output, or MIMO) systems which has no scalar (single-input, single-output, or SISO) counterpart is that of control input vector directionality. This problem is addressed in this paper by means of an optimal design of an artificial nonlinearity (AN).

The remainder of this paper is organized as follows. In Section 2, the parameterization of linear AW compensators is presented. Using this parametrization, conditions for controller implementability and closed-loop stability are derived. In Section 3, two AN designs are considered. In Section 4, a comparative simulation study of four AW designs for multivariable controllers is presented. Finally, some conclusions are drawn in Section 5.

2. *Parameterization of AW designs*

2.1. *Windup problem and AW compensation.* Let us consider the unity feedback, linear control system shown in Fig. 1, where the plant is described by a $m \times m$ linear transfer matrix $\mathbf{P}(s)$, and a linear controller with $m \times m$ transfer matrix $\mathbf{K}(s)$ has been designed to meet some performance specifications. Including the effect of input limitations leads to the system shown in Fig. 2, in which the block N is included to model the effects of input nonlinearities, and a distinction is now made between the real

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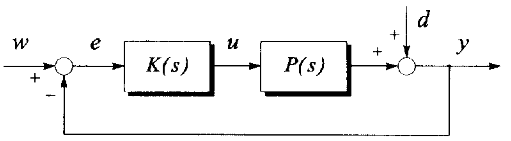

Fig. 1. Unity feedback control system.

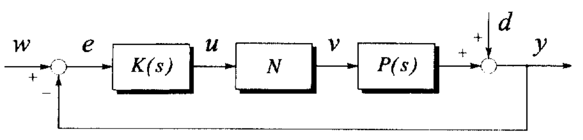

Fig. 2. Constrained unity feedback control system.

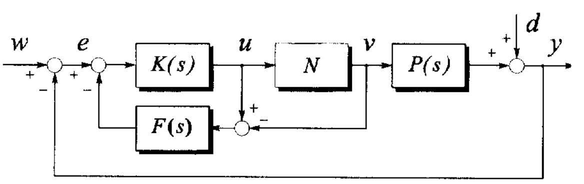

Fig. 3. Constrained unity feedback control system with linear AW compensator.

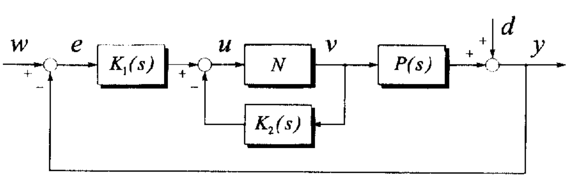

Fig. 4. Parameterization of linear AW compensators.

plant input v and the expected plant input $\mathbf{u}$, which is also the output of the linear controller $\mathbf{K}(s)$.

If a dynamical controller is updated without using any information of the real plant input $\mathbf{v}$ as shown in Fig. 2, the controller states will be incorrectly updated when $\mathbf{v}$ differs from $\mathbf{u}$, and this lack of consistency is referred to as controller windup. Since the design of $\mathbf{K}(s)$ ignores actuator nonlinearities, controller windup can result in significant performance degradation with respect to the expected linear performance.

In contrast, if the internal states of the dynamic controller $\mathbf{K}(s)$ are updated using knowledge of $\mathbf{v}$ when $\mathbf{v} \neq \mathbf{u}$, the controller implementation is said to incorporate AW compensation. It should be pointed out, however, that the mere presence of AW compensation is not sufficient to eliminate the degradation of the closed-loop performance (Vrančić and Peng, 1996a). Our objective, therefore, is to investigate which AW compensation should be used in which case. Appropriate AW design should lead to graceful degradation with respect to the expected linear performance when the control input enters saturation.

A natural way of using knowledge of $\mathbf{v}$ is to feed $(\mathbf{u} - \mathbf{v})$ back to the controller through a linear filter with an appropriate transfer matrix $\mathbf{F}(s)$ as shown in Fig. 3. As the AW compensation is achieved with a linear filter, AW designs of this type are referred to as linear AW compensators (Peng *et al.*, 1996). It is not a straightforward task to derive conditions for controller implementability and closed-loop stability for the scheme in Fig. 3. Another way to parametrize any linear AW compensator for unity feedback controller $\mathbf{K}(s)$ is shown in Fig. 4.

Indeed, by block manipulation, it can be easily shown that the schemes represented in Figs. 3 and 4 are equivalent if and only if

$$\mathbf{K}_1(s) = (\mathbf{I} + \mathbf{K}(s)\mathbf{F}(s))^{-1}\mathbf{K}(s), \quad (1)$$

$$\mathbf{K}_2(s) = -(\mathbf{I} + \mathbf{K}(s)\mathbf{F}(s))^{-1}\mathbf{K}(s)\mathbf{F}(s). \quad (2)$$

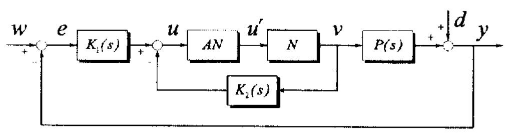

Fig. 5. AW schemes incorporating an artificial nonlinearity.

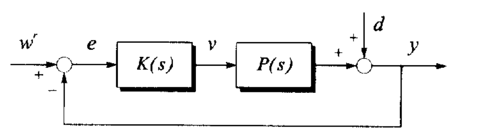

Fig. 6. Feedback control system with realizable reference $\mathbf{w}^r$.

Another possibility for modifying the AW scheme presented in Fig. 4 is shown in Fig. 5, where AN is an artificial nonlinearity chosen such that the output $\mathbf{u}^r$ never makes the nonlinearity N active, i.e. $\mathbf{v} = \mathbf{u}^r$ for all time. In practice, the block AN can only be added when block N is known *a priori*. The scheme in Fig. 4 can be therefore considered as a special case of the scheme shown in Fig. 5, corresponding to the case AN = **I**.

Kothare *et al.* (1994) have presented a unified framework for the study of anti-windup designs, in which all known anti-windup compensation schemes are shown to be a special cases. It should be pointed out that our proposed parametrization is equivalent to the framework of Kothare *et al.* (1994), but our interpretation is somewhat more intuitive, and provides some additional insights in the following manner. The anti-windup compensation framework of Kothare *et al.* can be abstracted as

$$\mathbf{u} = \mathbf{V}(s)\mathbf{e} + (\mathbf{I} - \mathbf{U}(s))\mathbf{v}, \quad (3)$$

where

$$\mathbf{V}(s) = \mathbf{H}_2 - \mathbf{H}_2\mathbf{C}(s\mathbf{I} - \mathbf{A} + \mathbf{H}_1\mathbf{C})^{-1}\mathbf{H}_1$$

$$\mathbf{U}(s) = \mathbf{H}_2\mathbf{D} + \mathbf{H}_2\mathbf{C}(s\mathbf{I} - \mathbf{A} + \mathbf{H}_1\mathbf{C})^{-1}(\mathbf{B} - \mathbf{H}_1\mathbf{D}). \quad (4)$$

A, **B**, **C**, and **D** are state-space matrices of $\mathbf{K}(s)$, and $\mathbf{H}_1$ and $\mathbf{H}_2$ are design parameter matrices of appropriate dimension. Indeed, expressions (3) and (4) are the state-space factorization of $\mathbf{K}(s)$. Noting $\mathbf{K}_1(s) = \mathbf{V}(s)$ and $\mathbf{K}_2(s) = \mathbf{U}(s)$-$\mathbf{I}$, we can see that our parametrization is the same as their framework.

However, as it will be shown in next sub-section, we found that $\mathbf{H}_2$ should not be used as a free design parameter, but should be fixed to $\mathbf{H}_2 = \mathbf{I}$. In such a case Kothare *et al.*'s framework coincides with the observer approach proposed by Åström and Wittenmark (1984).

It should be pointed out that the scheme represented by Fig. 4 is one of the simplest ways to parametrize any linear AW compensator for a unity feedback controller $\mathbf{K}(s)$. The reason is twofold. First, it is equivalent to the general framework given by Kothare *et al.* (1994) and therefore parametrizes any linear AW compensator. Second, a controller with an anti-windup compensator should at least contain two blocks, and this scheme does contain two blocks: $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$.

2.2. *Realizable reference.* For a given controller $\mathbf{K}(s)$, there are an infinite number of ways of assigning $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$. Likewise if N is known *a priori*, there are an infinite number of ways of designing AN. Our intention here is to investigate the pros and cons of several commonly advocated AW schemes. To this end, the concept of the realizable reference is introduced as follows. Assume that AN is fixed such that N is never active, i.e. $\mathbf{v} = \mathbf{u}^r$. It is always possible to make process input and output in Fig. 5 equivalent to those in Fig. 6 by suitable choice of reference $\mathbf{w}^r$. Thus different $\mathbf{K}_1(s)$, $\mathbf{K}_2(s)$ and AN will lead to different $\mathbf{w}^r$. If such a reference $\mathbf{w}^r$ were applied to the controller instead of the reference $\mathbf{w}$, the nonlinearity AN would not be active. For this reason, $\mathbf{w}^r$ is called the realizable reference (Hanus *et al.*, 1987).

From the viewpoint of tracking performance, the best AW strategy should make $\mathbf{w}^r$ as close as possible to $\mathbf{w}$. It can be easily deduced from Figs. 5 and 6 that

$$\mathbf{u} = \mathbf{K}_1(s)(\mathbf{w} - \mathbf{y}) - \mathbf{K}_2(s)\mathbf{v}, \quad (5)$$

$$\mathbf{v} = AN(\mathbf{u}) = \mathbf{K}_1(s)(\mathbf{w}^r - \mathbf{y}) - \mathbf{K}_2(s)\mathbf{v}. \quad (6)$$

These two equations lead to

$$\mathbf{w}^r = \mathbf{w} + \mathbf{K}_1^{-1}(s)(\mathbf{v} - \mathbf{u}). \quad (7)$$

For SISO systems, in order to make $\mathbf{w}^r$ as close as possible to $\mathbf{w}$, it is clear from equation (7) that AN should be the same as N, thereby ensuring $|\mathbf{v} - \mathbf{u}|$ is as small as possible. For this reason there is no need to introduce AN for SISO systems. However, for MIMO systems, it often happens that one actuator is saturated while the others are still working in linear regions. So it could be useful to use AN to modify $\mathbf{u}^r$ in order to achieve the desired realizable reference. This aspect will be addressed in details in Section 3.

2.3. *Conditions on* $\mathbf{K}_1(s)$ *and* $\mathbf{K}_2(s)$. The following conditions on $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$ must be met:

- C_1: $(\mathbf{I} + \mathbf{K}_2(s))^{-1}\mathbf{K}_1(s) = \mathbf{K}(s)$,
- C_2: $\mathbf{K}_2(s)$ is strictly proper,
- C_3: $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$ are stable.

Condition C_1, which can be derived from the equations (1) and (2), implies that in the absence of N, the controller remains the same as the nominal linear one. Condition C_2 implies that the controller is free of algebraic loops, i.e. can be implemented. Condition C_3 implies that whenever the plant input $\mathbf{v}$ is in saturation, the controller implementation is stable in the sense that the outputs of both $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$ are bounded. Note that no stability requirement is imposed on $\mathbf{K}(s)$.

One may argue that for continuous-time system, an algebraic loop can be tolerated and condition C_2 loses generality. For instance, in the general framework proposed by Kothare *et al.*, (1994), a design parameter $\mathbf{H}_2$ was proposed. It can be shown that condition C_2 is hold if and only if $\mathbf{H}_2 = \mathbf{I}$ ($\mathbf{I}$ is an identity matrix). In other words, if $\mathbf{H}_2 \neq \mathbf{I}$, condition C_2 will be violated. However, by investigating the following two theorems, we can find that $\mathbf{H}_2 \neq \mathbf{I}$ is of no use.

Theorem 1. Consider any two AW schemes of the form shown in Fig. 5, and applied to the same constrained closed-loop system with identical initial conditions. Denote these systems by AWa with $\mathbf{K}_1(s) = \mathbf{K}_{1a}(s)$ and $\mathbf{K}_2(s) = \mathbf{K}_{2a}(s)$ and AWb with $\mathbf{K}_1(s) = \mathbf{K}_{1b}(s)$ and $\mathbf{K}_2(s) = \mathbf{K}_{2b}(s)$. If $\mathbf{K}_{1b}(s) = \Gamma\mathbf{K}_{1a}(s)$ where Γ is any nonsingular matrix, then the realisable reference when using AWa is the same as that when using AWb.

Proof. See Appendix A.

Theorem 2. If $\mathbf{K}_{2a}(s)$ is proper but not strictly proper, there exists a non-singular matrix Γ such that $\mathbf{K}_{2b}(s)$ is strictly proper when setting $\mathbf{K}_{1b}(s) = \Gamma\mathbf{K}_{1a}(s)$.

Proof. See Appendix B.

Indeed, $\mathbf{H}_2 \neq \mathbf{I}$ leads to $\mathbf{K}_{2a}(s)$ which is proper but not strictly proper. From Theorem 1, it follows that using such a $\mathbf{K}_{2a}(s)$ is equivalent to using any $\mathbf{K}_{2b}(s)$ if it corresponds to $\mathbf{K}_{1b}(s) = \Gamma \cdot \mathbf{K}_{1a}(s)$, since the realizable references are the same for these two configurations. From Theorem 2, we can see that there exists a Γ such that $\mathbf{K}_{2b}(s)$ is strictly proper. On the other hand, a strictly proper $\mathbf{K}_{2b}(s)$ should correspond to $\mathbf{H}_2 = \mathbf{I}$. Therefore the imposition of $\mathbf{H}_2 = \mathbf{I}$ in condition C_2 implies no loss of generality, and the introduction of an algebraic loop by imposing $\mathbf{H}_2 \neq \mathbf{I}$ is of no use.

To analyze the stability of AW control system described by Fig. 5, only some sufficient conditions to guarantee closed-loop stability are known by using the techniques such as the Popov, Circle, and Off-axis Circle criteria, the passivity theorem and multiplier theory (Kothare and Morari, 1995). Note that the closed-loop AW control system contains a cascade link between a nonlinearity $\mathbf{N}$ and $\mathbf{P}(s)\mathbf{K}_1(s) + \mathbf{K}_2(s)$. We conjecture that stable $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$ lead to closed-loop systems having a greater chance of meeting the known sufficient conditions for stability, but a detailed investigation of this conjecture is a problem for further research.

2.4. *The choice of* $\mathbf{K}_1(s)$ *and* $\mathbf{K}_2(s)$. Let us first give some explicit meanings of the conditions C_1, C_2, and C_3. From C_1, we can obtain

$$\mathbf{K}_2(s) = \mathbf{K}^{-1}(s)\mathbf{K}_1(s) - \mathbf{I}. \quad (8)$$

Usually, a controller is biproper, and this implies that $\mathbf{K}(\infty)$ exists. In such case, from equation (8) and noting that $\mathbf{K}_2(s)$ should be strictly proper, we can deduce

- C_4: $\mathbf{K}_1(\infty) = \mathbf{K}(\infty)$ if $\mathbf{K}(\infty)$ exists.

However, it may happen that a controller is not biproper. For instance, if one insists on diagonally decoupling a square MIMO plant whose interactor matrix is non-diagonal, any unity feedback controller must be strictly proper. In this case, C_4 is no longer valid. To deduce a condition for replacing C_4, we introduce an interactor matrix $\xi(s)$ for $\mathbf{K}(s)$, such that

$$\xi(s)\mathbf{K}(s) = \bar{\mathbf{K}}(s) \quad (9)$$

$$\lim_{s\to\infty} \xi(s)\mathbf{K}(s) = \lim_{s\to\infty} \bar{\mathbf{K}}(s) = \bar{\mathbf{K}}(\infty) \quad (10)$$

where $\bar{\mathbf{K}}(\infty)$ is nonsingular. Please note that there are different ways to construct $\xi(s)$. From equation (9) and noting that $K_2(s)$ should be strictly proper, we can deduce

- C_5: $\mathbf{K}_1(s) = \xi^{-1}(s)\bar{\mathbf{K}}_1(s)$ and $\bar{\mathbf{K}}_1(\infty) = \bar{\mathbf{K}}(\infty)$.

Another potential problem arises when $\mathbf{K}(s)$ contains nonminimum phase zeros. For instance if one insists on diagonally decoupling a square nonminimum phase MIMO plants whose generalized interactor matrix is nondiagonal, any unity feedback controller must be non-minimum phase (Goodwin *et al.*, 1993; Weller, 1996). As $\mathbf{K}_2(s)$ is stable, from equation (8) we must ensure that $\mathbf{K}^{-1}(s)\mathbf{K}_1(s)$ is stable. The explicit condition for this can be derived using the concept of generalized interactor matrix. By generalized matrix, we mean that not only equations (9) and (10) are hold, but also $\xi(s)$ contains all the nonminimum phase zeros. Thanks to this generalization, C_5 is still valid since $\mathbf{K}^{-1}(s)\mathbf{K}_1(s)$ remains stable. The idea of using controller interactor matrix in the AW context was first proposed in Hanus and Peng (1991, 1992).

It should be pointed out that in practice, $\mathbf{K}(s)$ is usually biproper and has no nonminimum phase zeros. In this case, we propose to use the conditioning technique which sets

$$\mathbf{K}_1(s) = \mathbf{K}(\infty) \quad (11)$$

$$\mathbf{K}_2(s) = \mathbf{K}^{-1}(s)\mathbf{K}(\infty) - \mathbf{I}. \quad (12)$$

The reason for using the conditioning technique is twofold. First, as $\mathbf{K}_1(s)$ does not feedback any information of the real input $\mathbf{v}$, it should avoid the inclusion of any dynamics which may suffer from windup, as done in equation (11). The worst case is that $\mathbf{K}_1(s) = \mathbf{K}(s)$, i.e. $\mathbf{K}_1(s)$ contains all the dynamics of $\mathbf{K}(s)$. This case corresponds to no AW compensation. Second, from equation (7), it is clear that $\mathbf{w}^r$ becomes $\mathbf{w}$ at the instant when the controller leaves the limitation. Any dynamical $\mathbf{K}_1(s)$ will make $\mathbf{w}^r$ different from $\mathbf{w}$ after the controller leaves the limitation.

3. *Optimal AN design*

For multivariable control systems, the potential exists for some, but not all, plant inputs to enter saturation. In such a situation, a commonly advocated strategy is to scale down all controller inputs in such a way that the direction of $\mathbf{u}(t)$ the plant input vector is maintained (Campo and Morari, 1990). This is accomplished using the following AN design strategy, known as direction-preserving AN design:

$$\mathbf{u}^r(t) = AN(\mathbf{u}(t)) = \begin{cases} \mathbf{u}(t) & \text{if } \mathbf{u}(t) \text{ is in linear region} \\ \min\left\{\dfrac{\text{sat}(u_i(t))}{u_i(t)}\right\}\mathbf{u}(t) & \text{if } \mathbf{u}(t) \text{ enters the saturation} \end{cases}. \quad (13)$$

From equation (7), however, it is clear that such a strategy can easily make $\mathbf{w}^r$ far from $\mathbf{w}$. An alternative strategy has been proposed by Hanus and Kinnaert (1989), in which the artificial nonlinearity block AN is designed in such a way that $\mathbf{w}^r$ remains as close as possible to $\mathbf{w}$, in some sense. Since by definition, AN is no less limited than N, the resulting $\mathbf{w}^r$ is bounded if it is bounded without using AN, and thus the optimal AN design will not endanger closed-loop stability. While optimal AN design was originally proposed in (Hanus and Kinnaert, 1989), the presentation there is insufficiently detailed to permit ready implementation. For this reason, we choose to present a thorough treatment of optimal AN design in the present paper.

It should be emphasized that the optimal AN design is carried out after designing $\mathbf{K}_1(s)$ and $\mathbf{K}_2(s)$, so that design of AN is considered independent of any particular AW design. To simplify the presentation, the nonlinearity N is assumed to be restricted to the "*sat*" function, defined as

$$sat(\mathbf{u}) = [sat(\mathbf{u}_1) \cdots sat(\mathbf{u}_m)]^{\mathrm{T}}, \tag{14}$$

where

$$sat(u_i) = \begin{cases} u_i^{\max}, & u_i > u_i^{\max}, \\ u_i, & u_i^{\min} \le u_i \le u_i^{\max}, \\ u_i^{\min}, & u_i < u_i^{\min}. \end{cases} \tag{15}$$

Suppose that the conditioning technique (11)–(12) is used, and denote by $\mathbf{D}$ the nonsingular feedthrough matrix of the linear controller $\mathbf{K}(s)$, i.e. $\mathbf{D} = \mathbf{K}(\infty)$. Equation (7) can then be rewritten as

$$\mathbf{w}^r = \mathbf{w} + \mathbf{D}^{-1}(\mathbf{u}^r - \mathbf{u}). \tag{16}$$

By introducing $\mathbf{H} = [\mathbf{I}_{m\times m} \quad -\mathbf{I}_{m\times m}]^{\mathrm{T}}$ and $\mathbf{b} = [-u_1^{\max}, \ldots, -u_m^{\max}, u_1^{\min}, \ldots, u_m^{\min}]^{\mathrm{T}}$, the inequality constraints in equation (15) can be concisely represented as

$$h_u(\mathbf{u}^r) = \mathbf{H}\mathbf{u}^r + \mathbf{b} \le \mathbf{0}. \tag{17}$$

Using equation (16), the inequality constraints can be expressed in terms of $\mathbf{w}^r$ and $\mathbf{w}$:

$$h_w(\mathbf{w}^r) = \mathbf{HD}(\mathbf{w}^r - \mathbf{w}) + \mathbf{Hu} + \mathbf{b} \le 0. \tag{18}$$

Optimal AN design is therefore a nonlinear programming problem, the solution of which instantaneously minimizes a performance criterion $J(\mathbf{w}^r - \mathbf{w})$ subject to the inequality constraints in equation (18). By appropriate choice of $J(\bullet)$, both the criterion and constraint functions are convex. The optimal AN design problem therefore always has a solution which can be found via numerical techniques, e.g. the simplex method. However, since $\mathbf{u}$ is a time-varying signal, the optimal AN design should be carried out at every time instant, making such a solution computationally intensive. A closed-form solution first described by Hanus and Kinnaert (1989), and suitable for real-time applications is presented in Appendix C.

4. *Simulation results*

In previous sections, we have shown that the conditioning technique is usually a suitable AW technique and that optimal AN design can improve any AW technique. To support our theoretical development, we will compare the following four approaches, namely the original conditioning technique, the conditioning technique with direction preserving AN, the conditioning technique with optimal AN, and the internal model control (IMC) with AW compensation. For the last approach, we have chosen the modified IMC approach (Zheng *et al.*, 1994) since it was shown by Zheng *et al.* (1994) to be a superior solution to the original conditioning technique. Some simulations are performed to show the effectiveness of the proposed approach.

Consider the process described by

$$\mathbf{P}(s) = \frac{10}{1+100s}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}. \tag{19}$$

This process is used in (Zheng *et al.*, 1994). The classical IMC controller is given by $\tilde{\mathbf{P}}(s) = \mathbf{P}(s)$ and

$$\mathbf{Q}(s) = \frac{1+100s}{10(1+20s)}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}. \tag{20}$$

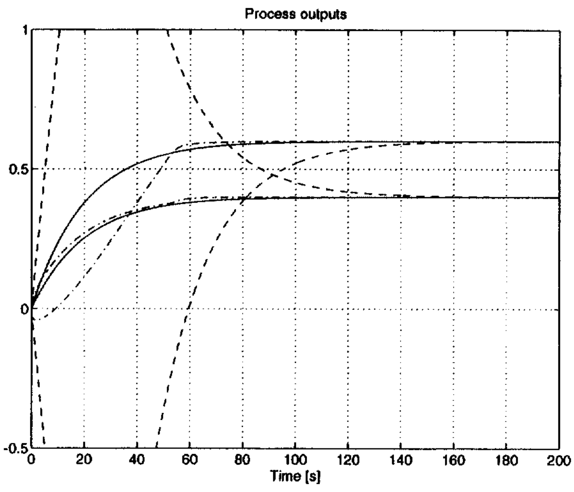

Fig. 7. The process outputs (y); —— unlimited system, - - original conditioning technique, -·- modified IMC approach.

For the meaning of $\tilde{\mathbf{P}}(s)$ and $\mathbf{Q}(s)$, please refer to (Zheng *et al.*, 1994). This controller is equivalent to a feedback controller with

$$\mathbf{K}(s) = \mathbf{Q}(\mathbf{I} - \tilde{\mathbf{P}}\mathbf{Q})^{-1} = \frac{1+100s}{200s}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}. \tag{21}$$

In the modified IMC approach, Zheng *et al.* (1994) worked with a modified plant model in order to achieve superior performance than the original conditioning technique. This implies that the controller too, was modified, with

$$\tilde{\mathbf{P}}(s) = \frac{10}{1+100s}\begin{bmatrix} 4 & -\dfrac{5}{0.1s+1} \\ -\dfrac{3}{0.1s+1} & 4 \end{bmatrix}, \tag{22}$$

and a filter $\mathbf{f} = 2.5(s+1)\mathbf{I}$ was introduced for the AW implementation. This leads to

$$\begin{aligned} \mathbf{K}_1(s) &= \mathbf{fPQ}, \\ \mathbf{K}_2(s) &= \mathbf{fP} - \mathbf{I} - \mathbf{fPQ}\tilde{\mathbf{P}} \end{aligned}. \tag{23}$$

To apply the conditioning technique, we take the feedback controller (21) and decompose it according to equations (11) and (12).

A set-point change of $[0.6\ 0.4]^{\mathrm{T}}$ is applied. Both inputs are constrained between the saturation limits ± 1. Figure 7 shows the process outputs for unlimited system, and a limited system when using the original conditioning technique and the modified IMC AW compensator. It can be clearly seen that the classical conditioning technique results in quite poor performance. The modified IMC approach results in improved performance. Figure 8 shows the process outputs obtained using the conditioning technique with those achieved by direction preserving AN, and the conditioning technique with optimal AN where the weighting factor was $\Lambda = \mathbf{I}$. It is clear that both approaches result in improved responses as compared with the original conditioning technique, even better than the modified IMC approach.

The following criteria can be used to compare the four AW design strategies:

$$J_1 = \sum_{i=1}^{2} \int_0^{\infty} |w_i^r(t) - w_i(t)|\,\mathrm{d}t, \tag{24}$$

$$J_2 = \sum_{i=1}^{2} \int_0^{\infty} (w_i^r(t) - w_i(t))^2\,\mathrm{d}t, \tag{25}$$

$$J_3 = \sum_{i=1}^{2} \int_0^{\infty} |y_{ul i}(t) - y_{\mathrm{AW}i}(t)|\,\mathrm{d}t, \tag{26}$$

$$J_4 = \sum_{i=1}^{2} \int_0^{\infty} (y_{ul i}(t) - y_{\mathrm{AW}i}(t))^2\,\mathrm{d}t, \tag{27}$$

where $y_{\mathrm{u}i}$ and $y_{\mathrm{AW}i}$ are the ith process output of the unlimited and limited process output with AW compensator, respectively.

The values of criteria functions are given in Table 1. It is obvious that the conditioning technique with optimal AN gives the best result.

Figures 9 and 10 show the effectiveness of the weighting factor Λ by taking

$$\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \tag{28}$$

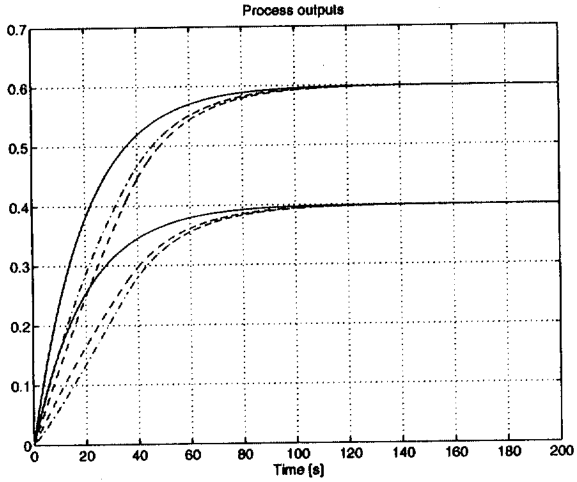

Fig. 8. The process outputs (**y**); — unlimited system, -- conditioning technique with direction preserving AN design, -·- conditioning technique with optimal AN design.

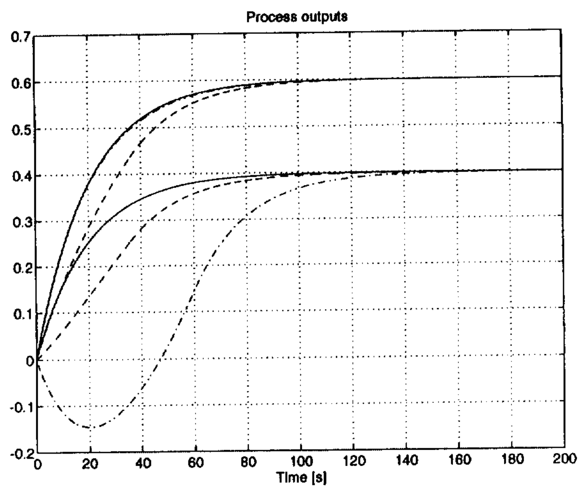

Fig. 9. Process outputs (**y**) when using conditioning technique with optimal AN design; — unlimited system, -- $\Lambda = \mathbf{I}$, -·- $\Lambda = [10\ 0; 0\ 1]$.

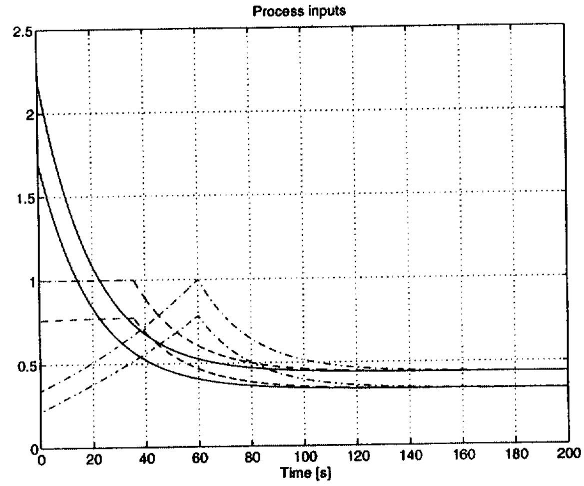

Fig. 10. Process inputs (**v**) when using conditioning technique with optimal AN design; — unlimited system, -- $\Lambda = \mathbf{I}$, -·- $\Lambda = [10\ 0; 0\ 1]$.

It is clear that, when using the conditioning technique with optimal AN, the unlimited and limited responses are quite similar for the first process output (at the cost of the greater differences between second one). Therefore, if the control of some process outputs must be tighter than that of the other process outputs, it can be achieved by changing the weighting factor Λ.

5. *Conclusions*

Linear AW techniques for multivariable controllers can easily be parametrized using the classical unity-feedback structure. Conditions for controller implementability and closed-loop stability can be stated in terms of this parametrization.

The case where the controller is not biproper (has non-minimum phase zeros) is also studied. It is shown that in such a case the controller interactor (generalized interactor) matrix can be introduced to derive explicit conditions for controller implementability and closed-loop stability. This also helps in explaining why the conditioning technique has some limitations and how these limitations can be removed.

It has been shown that AW designs for MIMO systems can be enhanced with the inclusion of an AN block, especially when mutual interactions between the process inputs and outputs are strong. As the conditioning technique is generally the most suitable AW technique for single-input single-output (SISO) systems (Peng *et al.*, 1996; Vrančić and Peng, 1996a; Vrančić *et al.*, 1996b), it seems that conditioning enhanced with an optimal AN design strategy is also a suitable AW technique for MIMO systems.

The practical implementation of the optimal AN has been presented, and the proposed algorithm, based on the Kuhn–Tucker theorem, appears to be simple and fast enough for real-time applications.

It is interesting to note that the conditioning technique has been implicitly implemented by many authors. For instance, Coulibaly *et al.* (1995) and Mhatre and Brosilow (1996)

Table 1. The values of criteria functions achieved by several AW techniques

	Original conditioning technique	Modified IMC with AW compensation	Conditioning technique with direction preserving AN	Conditioning technique with optimal AN
J_1	164.5	16.26	9.151	8.84
J_2	453.8	8.153	1.68	1.525
J_3	164.5	11.13	9.157	8.85
J_4	226.7	1.984	0.722	0.656

proposed the socalled new AW approach based on the IMC controller. However, by simple block manipulation, it can easily be shown that they used the conditioning technique as an AW protection. The reason for which they obtained improved response is that they changed the controller transfer function $K(s)$ when the system is saturated. Such a technique is not a general method and differs from the conventional AW compensations.

The advantage of using the conditioning technique as an AW technique is its simplicity: the resulting AW compensator depends only on the unconstrained controller transfer function, whereas most other approaches also require the process transfer function in order to calculate an appropriate AW compensator. Moreover, the robustness of AW compensators designed in this way when the identified process model is poor remains a topic for research.

It is sometimes claimed that a disadvantage of the conditioning technique is that it offers little design freedom in the form of additional tuning parameters. We would argue, however, that design freedom in the present context should be seen as a means of achieving satisfactory closed-loop performance, and that the freedom available in the design of the transfer matrix $K(s)$ and/or the additional artificial nonlinearity AN is almost always sufficient for this task.

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Appendix A—Proof of Theorem 1

For the sake of notation distinction, let us rewrite equations (5) and (7) for AWa and AWb respectively, i.e.

$$\mathbf{u}_a = \mathbf{K}_{1a}(s)(\mathbf{w} - \mathbf{y}) - \mathbf{K}_{2a}(s)\mathbf{v}, \quad (A.1)$$

$$\mathbf{w}_a^r = \mathbf{w} + \mathbf{K}_{1a}^{-1}(s)(\mathbf{v} - \mathbf{u}_a). \quad (A.2)$$

$$\mathbf{u}_b = \mathbf{K}_{1b}(s)(\mathbf{w} - \mathbf{y}) - \mathbf{K}_{2b}(s)\mathbf{v}, \quad (A.3)$$

$$\mathbf{w}_b^r = \mathbf{w} + \mathbf{K}_{1b}^{-1}(s)(\mathbf{v} - \mathbf{u}_b). \quad (A.4)$$

Note that due to the same initial conditions, $\mathbf{v}$ ($\mathbf{y}$) when using AWa is the same as $\mathbf{v}$ ($\mathbf{y}$) when using AWb at the beginning. If $\mathbf{w}_a^r$ and $\mathbf{w}_b^r$ are the same, then $\mathbf{v}$ ($\mathbf{y}$) when using AWa will be the same as $\mathbf{v}$ ($\mathbf{y}$) when using AWb.

From C_1, we can easily find that

$$\mathbf{K}_{2b}(s) = \mathbf{\Gamma}\mathbf{K}_{2a}(s) + \mathbf{\Gamma} - \mathbf{I}. \quad (A.5)$$

Combining equations (A.1), (A.3) and (A.5) yields

$$\mathbf{u}_b - \mathbf{v} = \mathbf{\Gamma}(\mathbf{u}_a - \mathbf{v}), \quad (A.6)$$

This leads to $\mathbf{w}_a^r = \mathbf{w}_b^r$ which concludes the proof.

Appendix B—Proof of Theorem 2

From condition C_1 and noting that $\mathbf{K}_1(s)$ and $\mathbf{K}(s)$ are proper, we deduce that $\mathbf{I} + \mathbf{K}_2(s)$ should be biproper. Hence $\mathbf{I} + \mathbf{K}_{2a}(\infty)$ is a non-singular matrix. From (A.5), we have thus

$$\mathbf{K}_{2b}(\infty) = \mathbf{\Gamma}(\mathbf{I} + \mathbf{K}_{2a}(\infty)) - \mathbf{I}. \quad (B.1)$$

Thus the choice $\mathbf{\Gamma} = (\mathbf{I} + \mathbf{K}_{2a}(\infty))^{-1}$ leads to $\mathbf{K}_{2b}(\infty) = \mathbf{0}$, i.e. $\mathbf{K}_{2b}(s)$ is strictly proper.

Appendix C—Explicit solution to the optimal AN design

In this appendix, we present an explicit solution to the optimal design of AN, the artificial nonlinearity block. For practical applications, it is reasonable to choose the quadratic weighting criterion for minimization:

$$J(\mathbf{w}^r - \mathbf{w}) = (\mathbf{w}^r - \mathbf{w})^T\mathbf{\Lambda}(\mathbf{w}^r - \mathbf{w}), \quad (C.1)$$

where $\mathbf{\Lambda}$ is a diagonal weighting matrix whose elements are all positive. The choice of $\mathbf{\Lambda}$ is dictated by the relative importance of each element of $\mathbf{w}^r - \mathbf{w}$ in the criterion. A Kuhn–Tucker multiplier $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_{2m}]^T \in \mathbf{R}^{2m}$ is introduced to form the following auxiliary Kuhn–Tucker function:

$$J^*(\mathbf{w}^r - \mathbf{w}) = (\mathbf{w}^r - \mathbf{w})^T\mathbf{\Lambda}(\mathbf{w}^r - \mathbf{w}) + \mu^T(\mathbf{HD}(\mathbf{w}^r - \mathbf{w}) + \mathbf{Hu} + \mathbf{b}) \quad (C.2)$$

where $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \ldots, \mathbf{H}_{2m}^T]^T \in \mathbf{R}^{2m \times m}$ and $\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{2m}]^T \in \mathbf{R}^{2m}$. In equation (C.2), $\mu_i = 0$ if the ith constraint is not active (i.e. if $\mathbf{H}_i\mathbf{D}(\mathbf{w}^r - \mathbf{w}) + \mathbf{H}_i\mathbf{u} + \mathbf{b}_i < 0$), and $\mu_i \geq 0$ if the ith constraint is active ($\mathbf{H}_i\mathbf{D}(\mathbf{w}^r - \mathbf{w}) + \mathbf{H}_i\mathbf{u} + \mathbf{b}_i = 0$). From the Kuhn–Tucker theorem, a first-order necessary condition for $\mathbf{w}^r - \mathbf{w}$ to be a local minimizer of J (a condition to be satisfied by any optimal AN design) is that

$$\frac{\partial J^*}{\partial \mathbf{w}^r} = 2\mathbf{\Lambda}(\mathbf{w}^r - \mathbf{w}) + \mathbf{D}^T\mathbf{H}^T\boldsymbol{\mu} = \mathbf{0} \quad (C.3)$$

If the controller output $\mathbf{u}$ violates no constraints (i.e. $\boldsymbol{\mu} = \mathbf{0}$), the optimal solution is $\mathbf{w}^r = \mathbf{w}$, in which case $\mathbf{u}^r = \mathbf{u}$. If, however, one or more elements of $\mathbf{u}$ violates the constraints (i.e. $\mathbf{H}_i\mathbf{u} + b_i > 0$ for some i), the optimal solution is found by setting $\mathbf{H}_i\mathbf{u}^r + b_i = 0$ for the same i. To this end, we partition $\boldsymbol{\mu}$, $\mathbf{H}$ and $\mathbf{b}$ as follows: $\boldsymbol{\mu} = [\boldsymbol{\mu}_0^T\ \boldsymbol{\mu}_+^T]^T$, $\mathbf{H} = [\mathbf{H}_+^T\ \mathbf{H}_0^T]^T$ and $\mathbf{b} = [\mathbf{b}_+^T\ \mathbf{b}_0^T]^T$, such that $\boldsymbol{\mu}_0 = \mathbf{0}$ contains the Kuhn–Tucker multipliers associated with the inactive constraints

$$\mathbf{H}_+\mathbf{D}(\mathbf{w}^r - \mathbf{w}) + \mathbf{H}_+\mathbf{u} + \mathbf{b}_+ < \mathbf{0}, \quad (C.4)$$

and $\boldsymbol{\mu}_+ > \mathbf{0}$ are the Kuhn–Tucker multipliers corresponding to the active constraints

$$\mathbf{H}_0\mathbf{D}(\mathbf{w}^r - \mathbf{w}) + \mathbf{H}_0\mathbf{u} + \mathbf{b}_0 = \mathbf{0}. \quad (C.5)$$

From equations (C.3) and (C.5) we obtain

$$\boldsymbol{\mu}_+ = 2(\mathbf{H}_0\mathbf{D}\mathbf{\Lambda}\mathbf{D}^T\mathbf{H}_0^T)^{-1}(\mathbf{H}_0\mathbf{u} + \mathbf{b}_0). \quad (C.6)$$

Note that since $\mathbf{H}_0$ picks out precisely those elements of $\mathbf{u}$ which violate the constraints (i.e. $\mathbf{H}_0\mathbf{u} + \mathbf{b}_0 > \mathbf{0}$), equation (C.6) implies $\boldsymbol{\mu}_+ \geq \mathbf{0}$. From equations (C.3) and (C.6), we obtain

$$\mathbf{w}^r - \mathbf{w} = -\mathbf{\Lambda}^{-1}\mathbf{D}^T\mathbf{H}_0^T(\mathbf{H}_0\mathbf{D}\mathbf{\Lambda}\mathbf{D}^T\mathbf{H}_0^T)^{-1}(\mathbf{H}_0\mathbf{u} + \mathbf{b}_0), \quad (C.7)$$

from which equation (16) yields

$$\mathbf{u}^r = -\mathbf{D}\mathbf{\Lambda}^{-1}\mathbf{D}^T\mathbf{H}_0^T(\mathbf{H}_0\mathbf{D}\mathbf{\Lambda}\mathbf{D}^T\mathbf{H}_0^T)^{-1}(\mathbf{H}_0\mathbf{u} + \mathbf{b}_0) + \mathbf{u}. \quad (C.8)$$

It can be readily verified that $\mathbf{u}^r$ computed as in equation (C.8) satisfies

$$\mathbf{H}_0\mathbf{u}^r + \mathbf{b}_0 = \mathbf{0}. \quad (C.9)$$

If this same $\mathbf{u}^r$ also satisfies

$$\mathbf{H}_+\mathbf{u}^r + \mathbf{b}_+ < \mathbf{0}, \quad (C.10)$$

then it is the unique optimal solution of the AN design problem. If equation (C.8) does not satisfy equation (C.10), then the corresponding line of $\mathbf{u}^r$ must be modified in order to find the optimal solution. Obtaining the appropriate modification is generally too complex a task for real-time applications. In practice, it is sufficient to apply $\mathbf{u}^r$ computed as in (C.8), in which case the nonlinearity N modifies some elements of $\mathbf{u}^r$ so that $\mathbf{H}_+\mathbf{v} + \mathbf{b}_+ \leq \mathbf{0}$. This method can therefore be considered a suboptimal solution to the AN design problem.