Anti-Windup Designs for Multivariable Controllers

YOUBIN PENG,† DAMIR VRANČIĆ,‡ RAYMOND HANUS† and STEVEN S. R. WELLERS§

Key Words—Control nonlinearities; multivariable control; saturation; anti-windup; direction preserving.

Abstract—This paper addresses two important aspects of anti-windup (AW) designs, namely the parametrization of linear AW compensators, and the role of artificial nonlinearity (AN) in the design of AW compensators for multivariable systems. For the first issue, a simple parametrization is given using the classical feedback structure in the framework of constrained unity-feedback multivariable control systems. For the second issue, two existing AN designs for coordinating plant inputs whenever one plant input enters saturation are reviewed. A comparative simulation study illustrates that the conditioning technique, enhanced by optimal AN design, gives the best tracking performance among different existing methods. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

Thermal control inputs for all industrial processes are subject to hard physical constraints. For instance, controllers may be constrained to produce outputs in the range 0–10 V or 0–20 mA, values cannot be opened more than 100% or less than 0%, motor-driven actuators have limited speeds, and so on. Such constraints are usually referred to as plant input limitations. Likewise, it is common practice to switch from manual to automatic mode, or between different controllers. Such mode switches are usually referred to as plant input substitutions.

As a result of limitations and substitutions, the real plant input may differ from the controller output. When this happens, the closed-loop performance may be significantly degraded in comparison with the expected performance of the controller designed to operate in a linear regime. This performance deterioration is referred to as windup. In the case of substitution, the difference between the outputs of different controllers may result in large jumps in the plant input, resulting in poor tracking performance. Mode switching leading to such phenomena is referred to as "bump transfer." One way of handling the problem of windup is to incorporate input limitations into the initial control design process. This approach can be quite involved, however, and the resulting (nonlinear) control laws can be very complicated. Moreover, the nonlinearities of the actuator are not always known a priori. A more common approach in practice is to perform a linear control design, then to add extra feedback compensation at the stage of control implementation. As this form of compensation aims to reduce the undesirable effects of windup, it is referred to as anti-windup (AW).

In the case of mode switching, methods aiming to minimize the jump at the plant input are referred to as bumpless transfer (BLT) methods. However, minimizing the jump in the manipulated variables is not always the best course of action; what is of primary importance is the resulting closed-loop tracking performance. Thus we refer to methods which not only reduce the jump at the plant input but also maintain acceptable tracking performance as conditioned transfer (CT) methods. AW strategies are usually treated as bumpless transfer methods, and indeed AW methods will usually reduce the jump at the plant input during mode switching. However, it should be pointed out that AW does not necessarily imply bumpless transfer. In fact, if AW aims at improving tracking performance during actuator saturation, then it will imply conditioned transfer in the case of mode switching.

The topic of AW design has been studied for some four decades by many authors. The most popular techniques are described in Harus (1989) and Kothare et al. (1994), and the references therein. In Kothare et al. (1994), a general framework for AW design is presented, in which all known AW compensation schemes arise as special cases.

Recently, we have provided guidelines for designing AW, BLT and CT compensation for PID controllers (Peng et al., 1996). Subsequent investigation has revealed that the conclusions can be extended to more general controllers. As a result, a key objective in this paper is to provide a simple parametrization for AW designs. Since the proposed parametrization is based on the classical feedback control scheme, it has some advantages over the framework proposed by Kothare et al. (1994). First, the AW design can be explained in a natural and straightforward manner. Second, different existing AW designs can easily be compared, and their advantages and disadvantages, readily understood.

One problem associated with saturation in multivariable (multi-input, multi-output, or MIMO) systems which has no scalar (single-input, single-output, or SISO) counterpart is that of control input vector directionality. This problem is addressed in this paper by means of an optimal design of an artificial nonlinearity (AN).

The remainder of this paper is organized as follows. In Section 2, the parametrization of linear AW compensators is presented. Using this parametrization, conditions for controller implementability and closed-loop stability are derived. In Section 3, two AN designs are considered. In Section 4, a comparative simulation study of four AW designs for multivariable controllers is presented. Finally, some conclusions are drawn in Section 5.

2. Parameterization of AW designs

2.1. Windup problem and AW compensation. Let us consider the unity feedback, linear control system shown in Fig. 1, where the plant is described by a \( m \times m \) linear transfer matrix \( P(s) \), and a linear controller with \( m \times m \) transfer matrix \( K(s) \) has been designed to meet some performance specifications. Including the effect of input limitations leads to the system shown in Fig. 2, in which the block \( N \) is included to model the effects of input nonlinearities, and a distinction is now made between the real

---

*Received 23 October 1996; revised 6 October 1997; received in final form 27 May 1998. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor E. Ydstie under the direction of Editor S. Skogestad. Corresponding author: Dr. Damir Vrančić. Tel. +386 61 1773 732, Fax +386 61 219 385, E-mail damir.vranic@ijs.si.
†Department of Control Engineering, Free University of Brussels, Avenue Franklin D. Roosevelt 50, B-1050, Brussels, Belgium.
‡Department of Computer Automation and Control, J. Stefan Institute, Jamova 39, 1001 Ljubljana, Slovenia.
§Department of Electrical and Computer Engineering, University of Newcastle, Callaghan NSW 2308, Australia.
Another possibility for modifying the AW scheme presented in Fig. 4 is shown in Fig. 5, where AN is an artificial nonlinearity chosen such that the output \( u' \) never makes the nonlinearity \( N \) active, i.e. \( v = u' \) for all time. In practice, the block AN can only be added when block \( N \) is known \( a \) priori. The scheme in Fig. 4 can be therefore considered as a special case of the scheme shown in Fig. 5, corresponding to the case \( \mathrm{AN} = \mathrm{I} \).

Kothare et al. (1994) have presented a unified framework for the study of anti-windup designs, in which all known anti-windup compensation schemes are shown to be special cases. It should be pointed out that our proposed parametrization is equivalent to the framework of Kothare et al. (1994), but our interpretation is somewhat more intuitive, and provides some additional insights in the following manner. The anti-windup compensation framework of Kothare et al. can be abstracted as

\[
\mathbf{u} = \mathbf{V}(s)\mathbf{w} + (I - \mathbf{U}(s))\mathbf{y},
\]

where

\[
\mathbf{V}(s) = \mathbf{H}_1 - \mathbf{H}_1\mathbf{C}(I - \mathbf{A} + \mathbf{H}_1\mathbf{C})^{-1}\mathbf{H}_1 \quad \mathbf{U}(s) = \mathbf{H}_2\mathbf{D} + \mathbf{H}_2\mathbf{C}(I - \mathbf{A} + \mathbf{H}_1\mathbf{C})^{-1}(\mathbf{B} - \mathbf{H}_1\mathbf{D})
\]

A, B, C, and D are state-space matrices of \( \mathbf{K}(s) \), and \( \mathbf{H}_1 \) and \( \mathbf{H}_2 \) are design parameter matrices of appropriate dimension. Indeed, expressions (3) and (4) are the state-space factorization of \( \mathbf{K}(s) \). Noting \( \mathbf{K}_1(s) = \mathbf{V}(s) \) and \( \mathbf{K}_2(s) = \mathbf{U}(s) \), we can see that our parametrization is the same as their framework.

However, as it will be shown in next sub-section, we found that \( \mathbf{H}_2 \) should not be used as a free design parameter, but should be fixed to \( \mathbf{H}_2 = \mathbf{I} \). In such a case Kothare et al.'s framework coincides with the observer approach proposed by Astrom and Wittenmark (1984).

It should be pointed out that the scheme represented by Fig. 4 is one of the simplest ways to parametrize any linear AW compensator for a unity feedback controller \( \mathbf{K}(s) \). The reason is twofold. First, it is equivalent to the general framework given by Kothare et al. (1994) and therefore parametrizes any linear AW compensator. Second, a controller with an anti-windup compensator should at least contain two blocks, and this scheme does contain two blocks: \( \mathbf{K}_1(s) \) and \( \mathbf{K}_2(s) \).

### 2.2. Realizable Reference

For a given controller \( \mathbf{K}(s) \), there are an infinite number of ways of assigning \( \mathbf{K}_1(s) \) and \( \mathbf{K}_2(s) \). Likewise if \( N \) is known \( a \) priori, there are an infinite number of ways of designing AN. Our intention here is to investigate the pros and cons of several commonly advocated AW schemes. To this end, the concept of the realizable reference is introduced as follows. Assume that AN is fixed such that \( N \) is never active, i.e. \( v = u' \). It is always possible to make process input and output in Fig. 5 equivalent to those in Fig. 6 by suitable choice of reference \( w' \). Thus different \( \mathbf{K}_1(s) \) and \( \mathbf{K}_2(s) \) and AN will lead to different \( w' \). If such a reference \( w' \) were applied to the controller instead of the reference \( w \), the nonlinearity AN would not be active. For this reason, \( w' \) is called the realizable reference (Hanus et al., 1987).

From the viewpoint of tracking performance, the best AW strategy should make \( w' \) as close as possible to \( w \). It can be easily deduced from Figs. 5 and 6 that

\[
\mathbf{u} = \mathbf{K}_3(s)(\mathbf{w} - \mathbf{y}) - \mathbf{K}_3(s)(\mathbf{w}' - \mathbf{y}) - \mathbf{K}_3(s)v,
\]

\[
v = A(N\mathbf{u}) = \mathbf{K}_3(s)(\mathbf{w}' - \mathbf{y}) - \mathbf{K}_3(s)v.
\]
These two equations lead to
\[ w' = w + K_1^{-1}(v - u). \]  
(7)

For SISO systems, in order to make \( w \) as close as possible to \( w \), it is clear from equation (7) that \( AN \) should be the same as \( N \), thereby ensuring \( |v - u| \) is as small as possible. For this reason there is no need to introduce \( AN \) for SISO systems. However, for MIMO systems, it often happens that one actuator is saturated while the others are still working in linear regions. So it could be useful to use \( AN \) to modify \( u \) in order to achieve the desired realizable reference. This aspect will be addressed in detail in Section 3.

### 2.3. Conditions on \( K_1(s) \) and \( K_2(s) \)

The following conditions on \( K_1(s) \) and \( K_2(s) \) must be met:

- \( C_1: T_1 + K_1^{-1}(s) = K(s) \),
- \( C_2: K_1(s) \) is strictly proper,
- \( C_3: K_1(s) \) and \( K_2(s) \) are stable.

Condition \( C_1 \), which can be derived from the equations (1) and (2), implies that in the absence of \( N \), the controller remains the same as the nominal linear one. Condition \( C_2 \) implies that the controller is free of algebraic loops, i.e., can be implemented. Condition \( C_3 \) implies that whenever the plant input \( v \) is in saturation, the controller implementation is stable in the sense that the outputs of both \( K_1(s) \) and \( K_2(s) \) are bounded. Note that no stability requirement is imposed on \( K(s) \).

One may argue that for continuous-time system, an algebraic loop can be tolerated and condition \( C_2 \) loses generality. For instance, in the general framework proposed by Kothare et al., (1994), design parameter \( H_1 \) was proposed. It can be shown that condition \( C_2 \) is hold if and only if \( H_1 = H \) (i.e., \( H \) is an identity matrix). In other words, if \( H_1 \neq 1 \), condition \( C_2 \) will be violated. However, by investigating the following two theorems, we can find that \( H_1 = 1 \) is of no use.

**Theorem 1.** Consider any two A/W schemes of the form shown in Fig. 5, and applied to the same constrained closed-loop system with identical initial conditions. Denote these systems by A/Wa with \( \mathbf{K}(s) = \mathbf{K}_a(s) \) and \( \mathbf{K}(s) = \mathbf{K}_b(s) \) and A/Wb with \( \mathbf{K}(s) = \mathbf{K}_a(s) \) and \( \mathbf{K}(s) = \mathbf{K}_b(s) \). If \( \mathbf{K}(s) = \mathbf{K}(s) \) where \( \mathbf{K} \) is any nonsingular matrix, then the realizable reference when using A/Wa is the same as that when using A/Wb.

**Proof.** See Appendix A.

**Theorem 2.** If \( K_2(s) \) is proper but not strictly proper, there exists a non-singular matrix \( \mathbf{K} \) such that \( \mathbf{K}(s) \) is strictly proper when setting \( \mathbf{K}_2(s) = \mathbf{K}_2(s) \).

**Proof.** See Appendix B.

Indeed, \( H_1 = 1 \) leads to \( K_2(s) \) which is proper but not strictly proper. From Theorem 1, it follows that using such a \( K_2(s) \) is equivalent to using any \( K_2(s) \) if it corresponds to \( K_2(s) = \mathbf{K}(s) \), since the realizable references are the same for these two configurations. From Theorem 2, we can see that there exists a \( \mathbf{K} \) such that \( \mathbf{K}(s) \) is strictly proper. On the other hand, a strictly proper \( K_2(s) \) should correspond to \( H_1 = 1 \). Therefore the imposition of \( H_1 = 1 \) in condition \( C_2 \) implies no loss of generality, and the introduction of an algebraic loop by imposing \( H_1 = 1 \) is of no use.

To analyze the stability of \( AW \) control system described by Fig. 5, only some sufficient conditions to guarantee closed-loop stability are known by using the techniques such as the Popov, Circle, and Off-axis Circle criteria, the passivity theorem and multiplier theory (Kothare and Morari, 1995). Note that the closed-loop \( AW \) control system contains a cascade link between a nonlinearity \( \mathbf{N} \) and \( \mathbf{P}(s)K(s) = \mathbf{K}(s) \). We conjecture that stable \( \mathbf{K}(s) \) and \( \mathbf{K}(s) \) lead to closed-loop systems having a greater chance of meeting the known sufficient conditions for stability, but a detailed investigation of this conjecture is a problem for further research.

**2.4. The choice of \( K_1(s) \) and \( K_2(s) \).** Let us first give some explicit meanings of the conditions \( C_1, C_2, \) and \( C_3 \). From \( C_1 \), we can obtain
\[ K_2(s) = K^{-1}(s)K_1(s) \]
(8)

Usually, a controller is bipoeroper, and this implies that \( K_2(s) \) exists. In such case, from equation (8) and noting that \( K_2(s) \) should be strictly proper, we can deduce
- \( C_2: K_1(s) = K_2(s) \) if \( K_2(s) \) exists.

However, it may happen that a controller is not bipoeroper. For instance, if one insists on diagonally decoupling a square MIMO plant whose interactator matrix is non-diagonal, any unity feedback controller must be strictly proper. In this case, \( C_2 \) is no longer valid. To deduce a condition for replacing \( C_2 \), we introduce an interactator matrix \( Q(s) \) for \( K(s) \), such that
\[ \xi(s)K(s) = \tilde{K}(s) \]
(9)
\[ \lim_{s \to \infty} \xi(s)K(s) = \lim_{s \to \infty} \tilde{K}(s) = \tilde{K_1}(s) \]
(10)
where \( \tilde{K}(s) \) is nonsingular. Please note that there are different ways to construct \( \tilde{K}(s) \). From equation (9) and noting that \( \tilde{K}(s) \) should be strictly proper, we can deduce
- \( C_2: K_1(s) = \tilde{Q}(s)K_2(s) \) and \( K_1(s) = K_2(s) \).

Another potential problem arises when \( K(s) \) contains nonminimum phase zeros. For instance, if one insists on diagonally decoupling a square nonminimum phase MIMO plants whose generalized interactator matrix is non-diagonal, any unity feedback controller must be non-minimum phase (Goodwin et al., 1993; Wellers, 1996). As \( K_2(s) \) is stable, from equation (8) we must ensure that \( K^{-1}(s)K_1(s) \) is stable. The explicit condition for this can be derived using the concept of generalized interactator matrix. By generalized matrix, we mean that not only equations (9) and (10) are hold, but also \( \xi(s) \) contains all the nonminimum phase zeros. Thanks to this generalization, \( C_2 \) is still valid since \( K^{-1}(s)K_1(s) \) remains stable. The idea of using controller interactator matrix in the AW context was first proposed in Hanus and Peng (1991, 1992).

It should be pointed out that in practice, \( K(s) \) is usually bipoeroper and has no nonminimum phase zeros. In this case, we propose to use the conditioning technique which sets
\[ K_1(s) = K(s) \]
(11)
\[ K_2(s) = K^{-1}(s)K(s) \]
(12)

The reason for using the conditioning technique is twofold. First, as \( K_2(s) \) does not feedback any information of the real input \( v \) it should avoid the inclusion of any dynamics which may suffer from windup, as done in equation (11). The worst case is that \( K_1(s) = K(s) \), i.e., \( K_1(s) \) contains all the dynamics of \( K(s) \). This case corresponds to no AW compensation. Second, from equation (7), it is clear that \( w' \) becomes \( w \) at the instant when the controller leaves the limitation. Any dynamical \( K_1(s) \) will make \( w' \) different from \( w \) after the controller leaves the limitation.

### 3. Optimal AN design

For multivariable control systems, the potential exists for some, but not all, plant inputs to enter saturation. In such a situation, a commonly advocated strategy is to scale down all controller inputs in such a way that the direction of \( u(t) \) the plant input vector is maintained (Cambo and Morari, 1990). This is accomplished using the following AN design strategy, known as direction-preserving AN design:

\[ u'(t) = AV(u(t)) \]
(1)
where
\[ u(t) = \begin{cases} \text{sat}(u(t)) & \text{if } u(t) \text{ is in linear region} \\ \text{sat}(u(t)) - u(t) & \text{if } u(t) \text{ enters the saturation} \end{cases} \]

From equation (7), however, it is clear that such a strategy can easily make \( w' \) far from \( w \). An alternative strategy has been proposed by Hanus and Kinnaird (1989), in which the artificial nonlinearity block AN is designed in such a way that \( w' \) remains as close as possible to \( w \), in some sense. Since by definition, AN is
no less limited than $N$, the resulting $w'$ is bounded if it is
bounded without using AN, and thus the optimal AN design
will not endanger closed-loop stability. While optimal AN
design was originally proposed in (Hanus and Kinnairn,
1989), the presentation there is insufficiently detailed to
permit ready implementation. For this reason, we choose to
present a thorough treatment of optimal AN design in the
present paper.

It should be emphasized that the optimal AN design is carried
out after designing $K_q(s)$ and $K_i(s)$, so that design of AN is
considered independent of any particular AW design. To sim-
plify the presentation, the nonlinearity $N$ is assumed to be
restricted to the "sat" function, defined as

$$\text{sat}(u) = [\text{sat}(u_1), \ldots, \text{sat}(u_n)]^T,$$

where

$$\text{sat}(u_i) = \begin{cases}
  u_{i_{\max}}, & u_i > u_{i_{\max}} \\
  u_i, & u_{i_{\max}} \leq u_i \leq u_{i_{\max}} \\
  u_{i_{\min}}, & u_i < u_{i_{\min}}.
\end{cases}$$

(14)

Suppose that the conditioning technique (11)–(12) is used, and
denote by $D$ the nonsingular feedthrough matrix of the linear
controller $K(s)$, i.e. $D = K(s)$. Equation (7) can then be
rewritten as

$$w' = w + D^{-1}(u - u).$$

(16)

By introducing $H = [L_{1}, \ldots, L_{n}]$ and $b = [-u_{1_{\max}}, \ldots, -u_{n_{\max}}, u_{1_{\min}}, \ldots, u_{n_{\min}}]^T$, the inequal-
ity constraints in equation (15) can be concisely represented as

$$h(w') = H w' + b \leq 0.$$

(17)

Using equation (16), the inequality constraints can be expressed
in terms of $w'$ and $w$.

$$h(w') = HD(w' - w) + H u + b \leq 0.$$  

(18)

Optimal AN design is therefore a nonlinear programming
problem, the solution of which instantaneously minimizes a per-
formance criterion $J(w') - w'$ subject to the inequality con-
straints in equation (18). By appropriate choice of $J(w')$, both the
criterion and constraint functions are convex. The optimal AN
design problem therefore always has a solution which can be
found via numerical techniques, e.g. the simplex method. How-
ever, since $u$ is a time-varying signal, the optimal AN design
should be carried out at every time instant, making such a solu-
tion computationally intensive. A closed-form solution first
described by Hanus and Kinnairn (1989), and suitable for real-time
applications is presented in Appendix C.

4. Simulation results
In previous sections, we have shown that the conditioning
 technique is usually a suitable AW technique and that optimal
 AN design can improve any AW technique. To support our
theoretical development, we will compare the following four
approaches, namely the original conditioning technique, the
conditioning technique with direction preserving AN, the condi-
tioning technique with optimal AN, and the internal model
control (IMC) with AW compensation. For the last approach,
we have chosen the modified IMC approach (Zheng et al., 1994)
since it was shown by Zheng et al. (1994) to be a superior
solution to the original conditioning technique. Some simu-
lations are performed to show the effectiveness of the proposed
approach.

Consider the process described by

$$P(s) = \frac{10}{1 + 100s} \begin{bmatrix}
  4 & -5 \\
  -3 & 4
\end{bmatrix}.$$  

(19)

This process is used in (Zheng et al., 1994). The classical IMC
gain is given by $P(s) = P(s)$ and

$$Q(s) = \begin{bmatrix}
  4 & 5 \\
  10(1 + 20s) & 3 & 4
\end{bmatrix}.$$  

(20)

Fig. 7. The process outputs ($y$), unlimited system, original conditioning technique, modified IMC approach.

For the meaning of $P(s)$ and $Q(s)$, please refer to (Zheng et al.,
1994). This controller is equivalent to a feedback controller with

$$K(s) = Q(I - \hat{P}Q)^{-1} = \begin{bmatrix}
  4 & -5 \\
  -3 & 4
\end{bmatrix}.$$  

(21)

In the modified IMC approach, Zheng et al. (1994) worked
with a modified plant model in order to achieve superior perform-
ance than the original conditioning technique. This implies
that the controller too, was modified, with

$$P(s) = \frac{10}{1 + 100s} \begin{bmatrix}
  4 & -5 \\
  -3 & 4
\end{bmatrix} \frac{5}{0.1s + 1}.$$  

(22)

and a filter $f = 2.5(s + 1)1$ was introduced for the AW
implementation. This leads to

$$K_1(s) = \hat{P}Q,$$

$$K_2(s) = P - \hat{P}Q \hat{P}^{-1}.$$  

(23)

To apply the conditioning technique, we take the feedback
controller (21) and decompose it according to equations (11)
and (12).

A set-point change of [0.6 0.4] is applied. Both inputs are
constrained between the saturation limits $\pm 1$. Figure 7 shows
the process outputs for unlimited system, and a limited system
when using the original conditioning technique and the modified
IMC AW compensator. It can be clearly seen that the classical
conditioning technique results in quite poor performance. The
modified IMC approach results in improved performance.
Figure 8 shows the process outputs obtained using the condi-
tioning technique with those achieved by direction preserving AN,
and the conditioning technique with optimal AN where the
weighting factor was $A = I$. It is clear that both approaches result
in improved responses as compared with the original condi-
tioning technique, even better than the modified IMC approach.

The following criteria can be used to compare the four AW
design strategies:

$$J_1 = \sum_{i=1}^{2} \int_{0}^{\infty} [w_i(t) - w_i(t)]^2 dt,$$

(24)

$$J_2 = \sum_{i=1}^{2} \int_{0}^{\infty} [w_i(t) - w_i(t)]^2 dt,$$

(25)

$$J_3 = \sum_{i=1}^{2} \int_{0}^{\infty} [y_i(t) - y_{AW}(t)]^2 dt,$$

(26)

$$J_4 = \sum_{i=1}^{2} \int_{0}^{\infty} [y_i(t) - y_{AW}(t)]^2 dt.$$  

(27)
where \( \chi_{\text{uni}} \) and \( \chi_{\text{AW}} \) are the \( i \)th process output of the unlimited and limited process output with AW compensator, respectively.

The values of criteria functions are given in Table 1. It is obvious that the conditioning technique with optimal AN gives the best result.

Figures 9 and 10 show the effectiveness of the weighting factor \( A \) by taking

\[
A = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}
\]  

(28)

Fig. 8. The process outputs (\( y \)): --- unlimited system, - conditioning technique with direction preserving AN design, --- conditioning technique with optimal AN design.

Fig. 9. Process outputs (\( y \)) when using conditioning technique with optimal AN design, --- unlimited system, --- \( A = I \), --- \( A = [10 \ 0 \ 0 \ 1] \).

Fig. 10. Process inputs (\( v \)) when using conditioning technique with optimal AN design; --- unlimited system, --- \( A = I \), --- \( A = [10 \ 0 \ 0 \ 1] \).

It is clear that, when using the conditioning technique with optimal AN, the unlimited and limited responses are quite similar for the first process output (at the cost of the greater differences between second one). Therefore, if the control of some process outputs must be tighter than that of the other process outputs, it can be achieved by changing the weighting factor \( A \).

5. Conclusions

Linear AW techniques for multivariable controllers can easily be parametrized using the classical unity-feedback structure. Conditions for controller implementability and closed-loop stability can be stated in terms of this parametrization.

The case where the controller is not biproper (has non-minimum phase zeros) is also studied. It is shown that in such a case the controller interact or (generalized interacter) matrix can be introduced to derive explicit conditions for controller implementability and closed-loop stability. This also helps in explaining why the conditioning technique has some limitations and how these limitations can be removed.

It has been shown that AW designs for MIMO systems can be enhanced with the inclusion of an AN block, especially when mutual interactions between the process inputs and outputs are strong. As the conditioning technique is generally the most suitable AW technique for single-input single-output (SISO) systems (Peng et al., 1996; Vrancic and Peng, 1996a; Vrancic et al., 1996b), it seems that conditioning enhanced with an optimal AN design strategy is also a suitable AW technique for MIMO systems.

The practical implementation of the optimal AN has been presented, and the proposed algorithm, based on the Kuhn-Tucker theorem, appears to be simple and fast enough for real-time applications.

It is interesting to note that the conditioning technique has been implicitly implemented by many authors. For instance, Coulby et al. (1995) and Mhatre and Brosilow (1996)

| Table 1. The values of criteria functions achieved by several AW techniques |
|---------------------------------|-----------------|-----------------|-----------------|
| Original conditioning technique | Modified IMC with AW compensation | Conditioning technique with direction preserving AN | Conditioning technique with optimal AN |
| \( J_1 \) | 164.5 | 16.26 | 9.151 | 8.84 |
| \( J_2 \) | 453.8 | 8.153 | 1.68 | 1.525 |
| \( J_3 \) | 164.5 | 11.13 | 9.157 | 8.85 |
| \( J_4 \) | 226.7 | 1.984 | 0.722 | 0.656 |
proposed the so-called new AW approach based on the IMC controller. However, by simple block manipulation, it can easily be shown that they used the conditioning technique as an AW protection. The reason for which they obtained improved response is that they changed the controller transfer function $K(s)$ when the system is saturated. Such a technique is not a general method and differs from the conventional AW compensations.

The advantage of using the conditioning technique as an AW technique is its simplicity: the resulting AW compensator depends only on the unconstrained controller transfer function, whereas most other approaches also require the process transfer function in order to calculate an appropriate AW compensator. Moreover, the robustness of AW compensators designed in this way when the identified process model is poor remains a topic for research.

It is sometimes claimed that a disadvantage of the conditioning technique is that it offers little design freedom in the form of additional tuning parameters. We would argue, however, that design freedom in the present context should be seen as a means of achieving satisfactory closed-loop performance, and that the freedom available in the design of the transfer matrix $K(s)$ and/or the additional artificial nonlinearity AN is almost always sufficient for this task.

Acknowledgements: The support to the first author for a visit to ETHZ, Switzerland, from the European Science Foundation, Scientific Programme on Control of Complex Systems (COSY), is acknowledged.

References


Appendix A — Proof of Theorem 1

For the sake of notation distinction, let us rewrite equations (5) and (7) for AWa and AWb respectively, i.e.

\[
\begin{align*}
\mathbf{u}_a &= K_{aw}(s)(w - \mathbf{y}) - K_{aw}(s)\mathbf{v}, \\
\mathbf{w}_a &= w + K_{aw}(s)\mathbf{v} - \mathbf{u}_a,
\end{align*}
\]

(4.1)

\[
\begin{align*}
\mathbf{u}_b &= K_{aw}(s)(w - \mathbf{y}) - K_{aw}(s)\mathbf{v}, \\
\mathbf{w}_b &= w + K_{aw}(s)\mathbf{v} - \mathbf{u}_b.
\end{align*}
\]

(4.2)

Note that due to the same initial conditions, $v(x)$ when using AWa is the same as $v(x)$ when using AWb at the beginning. If $\mathbf{w}_a$ and $\mathbf{w}_b$ are the same, then $v(x)$ when using AWb will be the same as $v(x)$ when using AWb.

From (4.1), we can easily find that

\[
K_{aw}(s) = \mathbf{1}K_{aw}(s)\mathbf{1} + \mathbf{I} - \mathbf{I}.
\]

(4.5)

Combining equations (4.1), (4.3) and (4.5) yields

\[
\mathbf{u}_a - v = \mathbf{v}(w - v),
\]

(4.6)

This leads to $\mathbf{w}_a = \mathbf{w}_b$, which concludes the proof.

Appendix B — Proof of Theorem 2

From condition $C_1$ and noting that $K_{aw}(s)$ and $K_{aw}(s)$ are proper, we deduce that $\mathbf{1} + K_{aw}(s)$ should be improper. Hence $\mathbf{1} + K_{aw}(s)$ is a non-singular matrix. From (4.5), we have thus

\[
K_{aw}(s) = \mathbf{1}\mathbf{1} + K_{aw}(s)\mathbf{1} - \mathbf{I}.
\]

(4.15)

Thus the choice $\mathbf{1} = (\mathbf{1} + K_{aw}(s)\mathbf{1})^{-1}$ leads to $K_{aw}(s) = 0$, i.e. $K_{aw}(s)$ is strictly proper.

Appendix C — Explicit solution to the optimal AN design

In this appendix, we present an explicit solution to the optimal design of AN, the artificial nonlinearity block. For practical applications, it is reasonable to choose the quadratic weighting criterion for minimization:

\[
J = \int (w' - w)^2 A(w' - w) + \mu [b' H(w' - w) + \mathbf{u} + \mathbf{b}]^2 d\mathbf{w}.
\]

(4.1)

where $A$ is a diagonal weighting matrix whose elements are all positive. The choice of $A$ is dictated by the relative importance of each element of $w' - w$ in the criterion. A Kuhn–Tucker multiplier $\mu = [\mu_1, \mu_2, \ldots, \mu_n]^T \in \mathbb{R}^n$ is introduced to form the following auxiliary Kuhn–Tucker function:

\[
J^* = \int \frac{1}{\hat{w}'(w' - w)^2 A(w' - w) + \mu^2 b' H(w' - w) + \mathbf{u} + \mathbf{b}]^2 d\mathbf{w}.
\]

C.2

where $\hat{w}' = [\hat{w}_1', \hat{w}_2', \ldots, \hat{w}_n']^T \in \mathbb{R}^n$ and $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T \in \mathbb{R}^n$. In equation (C.2), $\mu_i = 0$ if the ith constraint is not active (i.e. if $b' H(w' - w) + \mathbf{u} + \mathbf{b} < 0$), and $\mu_i \geq 0$ if the ith constraint is active (i.e. $(b' H(w' - w) + \mathbf{u} + \mathbf{b}) = 0$). From the Kuhn–Tucker theorem, a first-order necessary condition for $w' = w$ to be a local minimizer of $J$ (a condition to be satisfied by any optimal AN design) is that

\[
\frac{\partial J^*}{\partial \hat{w}'} = 2A(w' - w) + \mu^2 b' H^T \mu = 0
\]

(4.1)

If the controller output $\mathbf{u}$ violates no constraints (i.e. $\mu = 0$), the optimal solution is $w' = w$, in which case $\mathbf{w} = \mathbf{u}$, if, however, one or more elements of $\mathbf{u}$ violates the constraints (i.e. $\mathbf{u} + \mathbf{b} > 0$), the optimal solution is found by setting $\mathbf{w}' = \mathbf{u} + \mathbf{b} > 0$ for the same $i$. To this end, we partition $\mathbf{u}$ and $\mathbf{b}$ as follows: $\mu = [\mu_1^2, \mu_2^2, \ldots, \mu_n^2]$, $H = [H_1, H_2, \ldots, H_n]^T$ and $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T$, such that $\mu_i = 0$ contains the Kuhn–Tucker multipliers associated with the inactive constraints

\[
\mathbf{H}, \mathbf{D} \mathbf{w}' + \mathbf{u} + \mathbf{b} < 0,
\]

(4.3)
and $\mu_+ > 0$ are the Kuhn–Tucker multipliers corresponding to the active constraints

$$H_0^T D (w^* - w) + H_0 u + b_0 = 0.$$  \hfill (C.5)

From equations (C.3) and (C.5) we obtain

$$\mu_+ = 2 (H_0 D A D^T H_0^T)^{-1} (H_0 u + b_0).$$  \hfill (C.6)

Note that since $H_0$ picks out precisely those elements of $u$ which violate the constraints (i.e., $H_0 u + b_0 > 0$), equation (C.6) implies

$$\mu_+ \geq 0.$$  \hfill (C.7)

From equations (C.3) and (C.6), we obtain

$$w^* - w = - (I - D A D^T H_0^T)^{-1} (H_0 u + b_0).$$  \hfill (C.8)

from which equation (16) yields

$$u^* = - DA^{-1} (I - D A D^T H_0^T)^{-1} (H_0 u + b_0) + u.$$  \hfill (C.8)

It can be readily verified that $u^*$ computed as in equation (C.8) satisfies

$$H_0 u^* + b_0 = 0.$$  \hfill (C.9)

If this same $u^*$ also satisfies

$$H_0 u^* + b_0 < 0,$$  \hfill (C.10)

then it is the unique optimal solution of the AN design problem. If equation (C.8) does not satisfy equation (C.10), then the corresponding line of $u^*$ must be modified in order to find the optimal solution. Obtaining the appropriate modification is generally too complex a task for real-time applications. In practice, it is sufficient to apply $u^*$ computed as in (C.8), in which case the nonlinearity $N$ modifies some elements of $u^*$ so that $H_0 u^* + b_0 \leq 0$. This method can therefore be considered a suboptimal solution to the AN design problem.