Addendum and Correction to "Optimal Phases for a Family of Quadriphase CDMA Sequences"

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This correspondence presents several corrections and an addendum to the above paper[1].

<u>Correction 1:</u>

Equation (7) should read :

$$\sum_{l=0}^{L-1} |C(x,y)(l)|^2 + \sum_{l=0}^{L-1} |C(x,y)(l-L)|^2 = \sum_{l=0}^{L-1} C(x,x)(l)[C(y,y)(l)]^* + \sum_{l=0}^{L-1} C(x,x)(l-L)[C(y,y)(l-L)]^*. \qquad (7)$$

<u>Correction 2:</u>

There is an error in the second equation following (19). The revised text should read:

In view of (13), the above is actually equal to

$$\sum_{l=1}^{L-2} (L+1)(L-l) = (L+1)^2(L-2)/2.$$

Similarly, we can show that

$$\sum_{l=1}^{L-2} \sum_{x \in U_\alpha} |C(x,x)(l+1)|^2 = (L^2-1)(L-2)/2.$$

Thus the right-hand side of (19) is equal to $L(L+1)(L-2)$. Substituting these results into (17), we have the average user interference

$$\frac{1}{A} \frac{\binom{L-1}{A-2}}{\binom{L+1}{A}} \sum_{y \in U_\alpha} \sum_{x \in U_\alpha; x \neq y} (6L^3)^{-1} [2\mu_{x,y}(0) + \mathrm{Re}\{\mu_{x,y}(1)\}] \leq \frac{A-1}{3L} \left(1 - \frac{1}{2L}\right) \qquad (20)$$

[1]F.-W. Sun and H. Leib, "Optimal phases for a family of quadriphase CDMA sequences", *IEEE Trans. Inform. Theory*, vol. 43, no. 4, pp. 1205–1217, July 1997.

and

$$\frac{1}{A}\frac{\binom{L-1}{A-2}}{\binom{L+1}{A}}\sum_{y\in U_\alpha}\sum_{x\in U_\alpha;x\neq y}(6L^3)^{-1}[2\mu_{x,y}(0)+\mathrm{Re}\{\mu_{x,y}(1)\}] \geq \frac{A-1}{3L}\left(1-\frac{3L-4}{2L^2}\right) \tag{21}$$

when there are A active users out of $L+1$ possible users. The difference between the upper and lower bounds is only $(A-1)(L-2)/3L^3$.
Substituting the upper and lower bounds of (20) and (21) into (18) leads, respectively, to the lower bound on the average signal-to-noise ratio

$$\left\{\frac{A-1}{3L}\left(1-\frac{1}{2L}\right)+N_0/2E_b\right\}^{-1/2} \tag{22}$$

and the upper bound

$$\left\{\frac{A-1}{3L}\left(1-\frac{3L-4}{2L^2}\right)+N_0/2E_b\right\}^{-1/2}. \tag{23}$$

Correction 3:

The expression for the average user interference with ideal random sequences from [22] that is used in the above paper[1] after equation (23) is incorrect[2]. The correct expression is the one from [13]

$$\frac{A-1}{3L}$$

that in fact improves the results from the above paper[1].

Correction 4:

As a consequence of the corrected bounds (22)–(23), the values of several numerical quantities in Section VII need revising. Equation (42) and its successor should read:

$$\{0.04422\cdot(A-1)+N_0/2E_b\}^{-1/2} \tag{42}$$

and

$$\{0.03936\cdot(A-1)+N_0/2E_b\}^{-1/2}.$$

Likewise, the text appearing immediately under Fig. 5 should read:

whereas the lower and upper bounds of (22) and (23) are, respectively,

$$\{0.02148\cdot(A-1)+N_0/2E_b\}^{-1/2}$$

and

$$\{0.02020\cdot(A-1)+N_0/2E_b\}^{-1/2}.$$

[2]D.V. Sarwate, "Comments on "An alternative derivation for the signal-to-noise ratio of a SSMA system" ", *IEEE Trans. Commun.*, vol. 43, no. 12, p. 2903, Dec. 1995.

Correction 5:

In view of the correction to the expression for the average user interference with ideal random sequences, the corresponding numerical result from equation (43) should read

$$\{0.047619 \cdot (A-1) + N_0/2E_b\}^{-1/2}, \qquad (43)$$

and the second equation after (44) should read

$$\frac{1}{\sqrt{0.047619(A-1)}}.$$

The fifth equation after (44) that gives the largest achievable gain of the sequences from Table I with respect to random sequences should read

$$10 \log(0.047619/0.04123) = 0.63 \text{ dB},$$

whereas the subsequent equation that gives the loss of the sequences from Table II with respect to random sequences should read

$$10 \log(0.055185/0.047619) = 0.64 \text{ dB}.$$

Addendum:

The average user interference specified by equation (17) in Section V is over all the cardinality A subsets U of U_α, implying that U_α contains such a subset with average user interference not larger than equation (17). Now, there is a need to compare (17) with the average user interference of random sequences. In the absence of an explicit expression for (17), the above paper[1] presents the upper and lower bounds (20) and (21), that yield corresponding lower and upper bounds to signal-to-noise ratio given by (22) and (23). Therefore:

1. The set U_α contains a subset U with average user interference not larger than the upper bound of (20). This upper bound is less than $(A-1)/3L$, the average user interference of random sequences. A similar result for Gold binary sequences is known[3].

2. The difference between the upper and lower bounds is $(A-1)(L-2)/3L^3$, showing that (20) is actually an increasingly tight upper bound to (17) with increasing L.

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[3] D.V. Sarwate, "Mean-square correlation of shift-register sequences", *Proc. IEE*, Part F, vol. 131, no. 2, pp. 101–106, April 1984.