A family of biphase CDMA sequences satisfying the Welch bound equality

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Introduction

In code-division multiple access (CMDA) communication systems, a number of users simultaneously transmit information over a common channel using different code sequences referred to as signature sequences. To achieve a low level of multiple access interference (MAI) in asynchronous CDMA systems, the signature sequences that are assigned to users need to have low cross-correlations for all relative time delays, while each sequence is required to have small autocorrelation sidelobes so as to enable acquisition and synchronization. Lower bounds on maximum nontrivial periodic correlations for CDMA signature sets have been proposed by Welch [1] and Sidelnikov among others, and numerous of families of signature sets have been proposed which are good (or even optimal) with respect to these criteria, including Gold, Kasami, bent, GMW and No sequences.

Recently, however, the immediate practical relevance of minimising the maximum nontrivial periodic correlation to performance (and thus capacity) of CDMA-based cellular systems has been called into question for two reasons. First, the most meaningful performance measure of a CDMA system, namely the average bit-error rate (BER), is much more closely associated with the average MAI than the worst-case value. Second, in biphase systems where the duration of one data bit equals the spreading sequence period and the data sequence is random, the odd correlation function is equally important as the even (or periodic) correlation function. Despite this, for the most part it is only the spectra of the even correlations of signature sets which have been reported in the literature. The explanation for this is that the odd (or polyphase) correlations depend on the phases of the signature sequences, whereas the even correlations do not, thereby significantly complicating the analysis. Recently, a direct generalization of Welch's basic bound [1], termed the *Welch bound equality (WBE)*, has been proposed which incorporates odd (or, more generally, polyphase) correlations; see [2] for details.

In this paper, we propose as a family of biphase CDMA sequences the coefficients of the sequence of polynomials $\{P_{n,m}(z)\}, n \geq 0, 0 < m \leq 2^n$ recently presented by Byrnes [3], which satisfy the Welch bound equality since the Byrnes matrix is a Hadamard matrix. Preliminary numerical results indicate that the Byrnes set exhibits an overall performance gain in worst-case (even and odd) autocorrelation sidelobe performance as compared to Gold sequences, so that synchronization and multipath performance is improved, while simultaneously achieving smaller average user interference (and hence BERs) than that of the Gold and ideal random signature sequences of comparable size.

The Welch bound equality (WBE)

A description of the model employed for the asynchronononous direct sequence CDMA communication system is given in [2]. Let X denote an ensemble of K signature sequences of length L whose entries are M-th roots of unity. Then the polyphase correlations for two sequences $x = \{x_i; i = 0, 1, \cdots, L-1\}$ and $y = \{y_i; i = 0, 1, \cdots, L-1\}$ are defined by $\theta^{(\gamma)}(x, y)(l) = C(x, y)(l) + \gamma C(x, y)(l - L)$ for γ an M-th root of unity and $C(x, y)$ the aperiodic correlation of x and y. If $M = 2$, X is said to be a biphase (or binary) signature set, and $\theta^{(1)}(x, y)$ and $\theta^{(-1)}(x, y)$ are termed the even and odd correlations respectively. The mean-square polyphase correlation satisfies the following inequality for each M-th root of unity γ [2]:

$$\sum_{x,y\in X}\sum_{l=0}^{L-1}|\theta^{(\gamma)}(x,y)(l)|^2 \geq L^2K^2. \tag{1}$$

If the equality holds for every γ, the signature set is said to satisfy the Welch bound equality, and thereby possesses desirable characteristics in terms of average user interference, signal-to-noise ratio (SNR) and BERs; see [2] for details.

Byrnes sequences

This biphase family is constructed inductively; see [3]. Let $P_{1,1}(z) = 1+z, P_{1,2}(z) = 1-z$. Given $P_{n,m}(z), m = 1,2,\ldots,2^n$, for $j = 0,1,2,\ldots,2^{n-1}-1$ and $m = 4j+1$ define $P_{n+1,m}(z), m = 1,2,\ldots,2^{n+1}$ by

$$\begin{aligned}
P_{n+1,m} &= P_{n,2j+1} + z^{2^n}P_{n,2j+2}\\
P_{n+1,m+1} &= P_{n,2j+1} - z^{2^n}P_{n,2j+2}\\
P_{n+1,m+2} &= P_{n,2j+2} + z^{2^n}P_{n,2j+1}\\
P_{n+1,m+3} &= P_{n,2j+2} - z^{2^n}P_{n,2j+1}.
\end{aligned}$$

The following table compares key parameters of a 32×32 Byrnes signature set with a set of Gold codes of comparable size obtained from the generating polynomial $g(x) = x^{10}+x^9+x^6+x^5+x^4+x^2+1$. In each case the maximum possible number of users is assumed. Note how the average multiple user interference is smaller for the Byrnes signature set than for the Gold codes or even ideal random signatures, while the even and odd autocorrelation sidelobe magnitude figures suggest that this improvement in average performance is made without compromising synchronization or multipath performance.

signature set	size of set/WBE	m.s. polyphase correlation		largest autocorrelation sidelobe		average user interference
		even	odd	even	odd	
Gold	33 × 31/1,046,529	1,046,559	1,047,295	9	17	0.3335
ideal random	32 × 32/1,048,576	—	—	—	—	0.3204
Byrnes	32 × 32/1,048,576	1,048,576	1,048,576	12	10	0.3193

References

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