Model reference adaptive control with hysteresis switching

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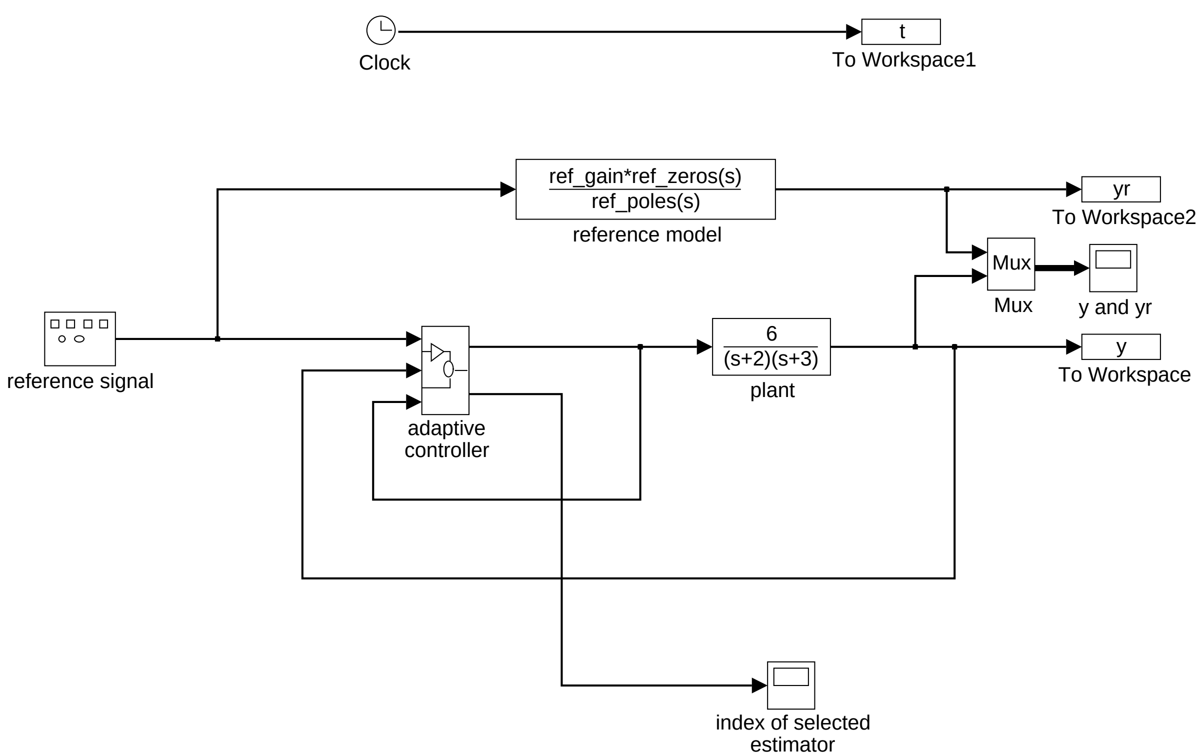

1. Introduction and background

- **Plant assumptions**

The plant is a single-input, single-output (SISO), linear, time-invariant (LTI) system, described by a transfer function

$$\frac{y(s)}{u(s)} = P(s) = k\frac{n_{\mathrm{p}}(s)}{d_{\mathrm{p}}(s)},$$

where

- $n_{\mathrm{p}}(s)$, $m_{\mathrm{p}}(s)$ are monic, coprime polynomials
- $\partial n_{\mathrm{p}}(s) = n$, $\partial m_{\mathrm{p}}(s) = m$
- the plant is strictly proper (relative degree $d = n - m > 0$), and minimum phase
- the sign of the high-frequency gain k is known

- **Reference model assumptions**

- The reference model is described by

$$\frac{y^*(s)}{r(s)} = \mathcal{M}^{-1}(s) = k_m\frac{n_{\mathrm{m}}(s)}{d_{\mathrm{m}}(s)}$$

- **Fixed control law**

When the plant transfer function is *known*, we can use a controller structure of the following form to obtain an asymptotically stable closed-loop system such that the plant output y tracks reference model output y^*:

$$\Delta[y^*] = Ru + \tilde{W}\Lambda^{-1}[y] + \tilde{S}\Lambda^{-1}[u],$$

or, equivalently,

$$u = \frac{1}{R}\left(\Delta\mathcal{M}^{-1}[r] - \tilde{S}\Lambda^{-1}[u] - \tilde{W}\Lambda^{-1}[y]\right)$$

where

- $\Delta(s) = (s + a)^d$
- R is a non-zero scalar
- $\tilde{W}$ and $\tilde{S}$ are polynomials in s

Example

$$\begin{aligned} P(s) &= \frac{6}{(s+2)(s+3)} & \text{plant} \\ \mathcal{M}^{-1}(s) &= \frac{16}{(s+4)^2} & \text{reference model} \end{aligned}$$

It can be shown that with $\Lambda(s) = (s+5)^2$,

$$R = 6, \qquad \tilde{W}(s) = 33s^2 + 174s + 225, \qquad \tilde{S}(s) = 6s - 150$$

achieves model matching:

$$\frac{y(s)}{r(s)} = \mathcal{M}^{-1}(s)$$

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• Estimation of controller coefficients

• When the coefficients of the plant transfer function are *unknown*, estimate R and the coefficients of $\tilde{W}, \tilde{S}$ with an identification scheme.

• It can be shown that the plant input-output behaviour can be described in the following form:

$$L^{-1}\Delta[y] = RL^{-1}[u] + \tilde{W}(\Lambda L)^{-1}[y] + \tilde{S}(\Lambda L)^{-1}[u],$$

or, in regression form, as

$$L^{-1}\Delta[y] = \theta^T \psi$$

where

- $L(s)$ is monic, Hurwitz, and $\partial L(s) = d$
- θ contains R and coefficients of $\tilde{W}$ and $\tilde{S}$ polynomials
- the regression vector ψ contains proper filtered derivatives of u and y
- Now use the "identification error"

$$e = L^{-1}\Delta[y] - \hat{\theta}^T \psi$$

to drive a normalized gradient algorithm for updating estimates $\hat{\theta}$ of θ:

$$\dot{\hat{\theta}} = \frac{\psi e}{1 + \psi^T \psi}$$

Example (cont.)

With $L(s) = (s+1)^2$ and $\Delta(s) = (s+3)^2$,

$$L^{-1}\Delta[y] = \theta^T \psi$$

where

$$\begin{aligned} \theta^T &= \left[\, R \;\; \tilde{W}_2 \;\; \tilde{W}_1 \;\; \tilde{W}_0 \;\; \tilde{S}_1 \;\; \tilde{S}_0 \,\right] \\ &= \left[\, 6 \;\; 33 \;\; 174 \;\; 225 \;\; 6 \;\; -150 \,\right] \end{aligned}$$

and

$$\begin{aligned} \psi^T &= \left[\, L^{-1}[u] \;\; s^2(\Lambda L)^{-1}[y] \;\; s(\Lambda L)^{-1}[y] \;\; (\Lambda L)^{-1}[y] \;\; s(\Lambda L)^{-1}[u] \;\; (\Lambda L)^{-1}[u] \,\right] \\ &= \left[\, \tfrac{[u]}{(s+1)^2} \;\; \tfrac{s^2[y]}{(s+5)^2(s+1)^2} \;\; \tfrac{s[y]}{(s+5)^2(s+1)^2} \;\; \tfrac{[y]}{(s+5)^2(s+1)^2} \;\; \tfrac{s[u]}{(s+5)^2(s+1)^2} \;\; \tfrac{[u]}{(s+5)^2(s+1)^2} \,\right] \end{aligned}$$

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- **Model reference adaptive control**

• Aggregating the fixed control law and the recursive parameter identification scheme—using the estimated parameters as if these were the true parameters—yields a *certainty equivalence* adaptive controller.

• Proving closed-loop stability for such a system is a non-trivial task, and requires the following assumptions concerning the plant:

1. the sign of the high-frequency gain k is known
2. an upper bound on the plant order n is known
3. the relative degree $d = n - m$ is known
4. the plant is minimum phase (i.e. $n_{\mathrm{p}}(s)$ is Hurwitz)

• These conditions are highly restrictive.

1. Are the conditions intrinsic to the problem?
2. Are less restrictive and/or more realistic assumptions sufficient to prove stability?

2. Hysteresis switching adaptive control of SISO plants

• Once the control structure is fixed, the class of plants able to be controlled is also fixed by:

- the sign of the high-frequency gain (i.e. sign of R)
- the admissible plant relative degree (i.e. by $\Delta(s) = (s + a)^d$)

• **Idea**

Broaden the class of admissible plants (and therefore weaken the assumptions) by defining a number of estimators, each of which has associated with it a class of plants which it can adaptively control

• Only 2 problems remain:

1. which is the right estimator?
2. only one controller can be connected to the plant at any given time

• **Analogy**

There are only 2 problems with winning Lotto:

1. how to pick the winning numbers
2. what to do with the money when you win

- **Another idea**

Get each estimator to give an indication of how well it is doing by producing a *test function*, e.g.

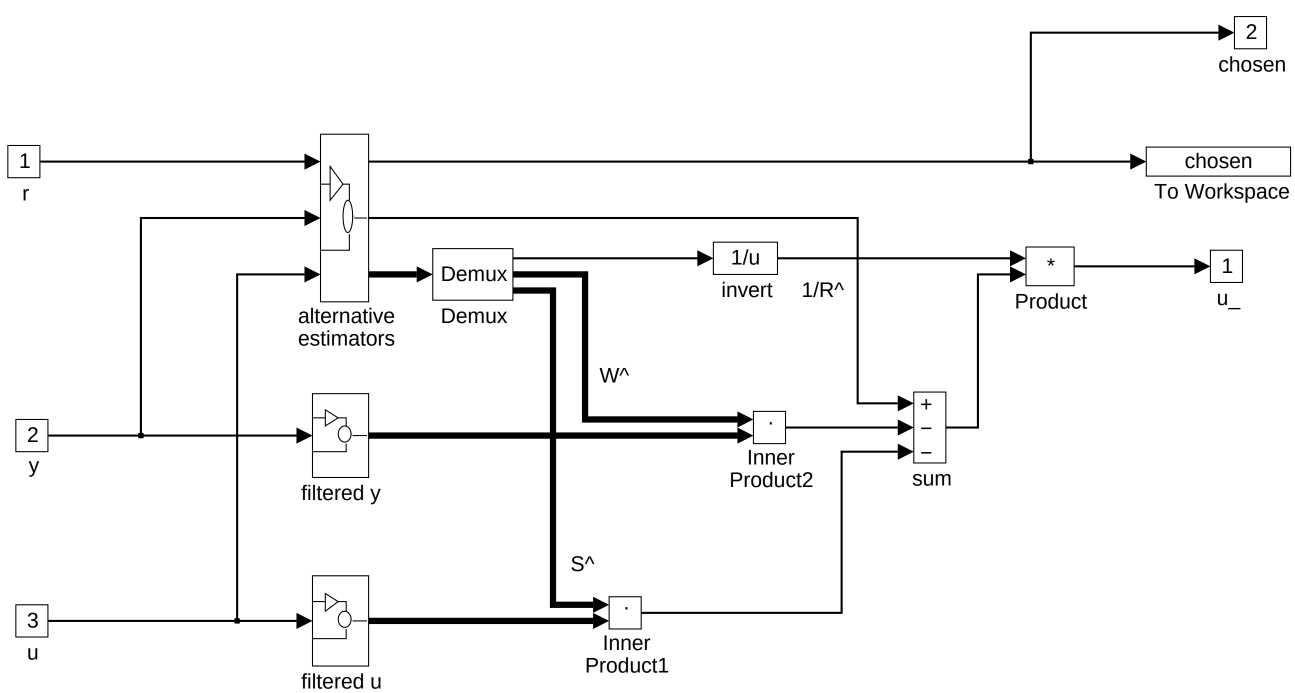
$$\delta_i(t) = \int_0^t \tilde{e}_i(\tau)^2 d\tau$$

where $\tilde{e}_i$ is the normalized "prediction error" associated with the ith estimator:

$$\tilde{e}_i = \frac{e_i}{(1+\psi^T\psi)^{1/2}}$$

$$e_i = L^{-1}\Delta[y] - \hat{\theta}_i^T\psi$$

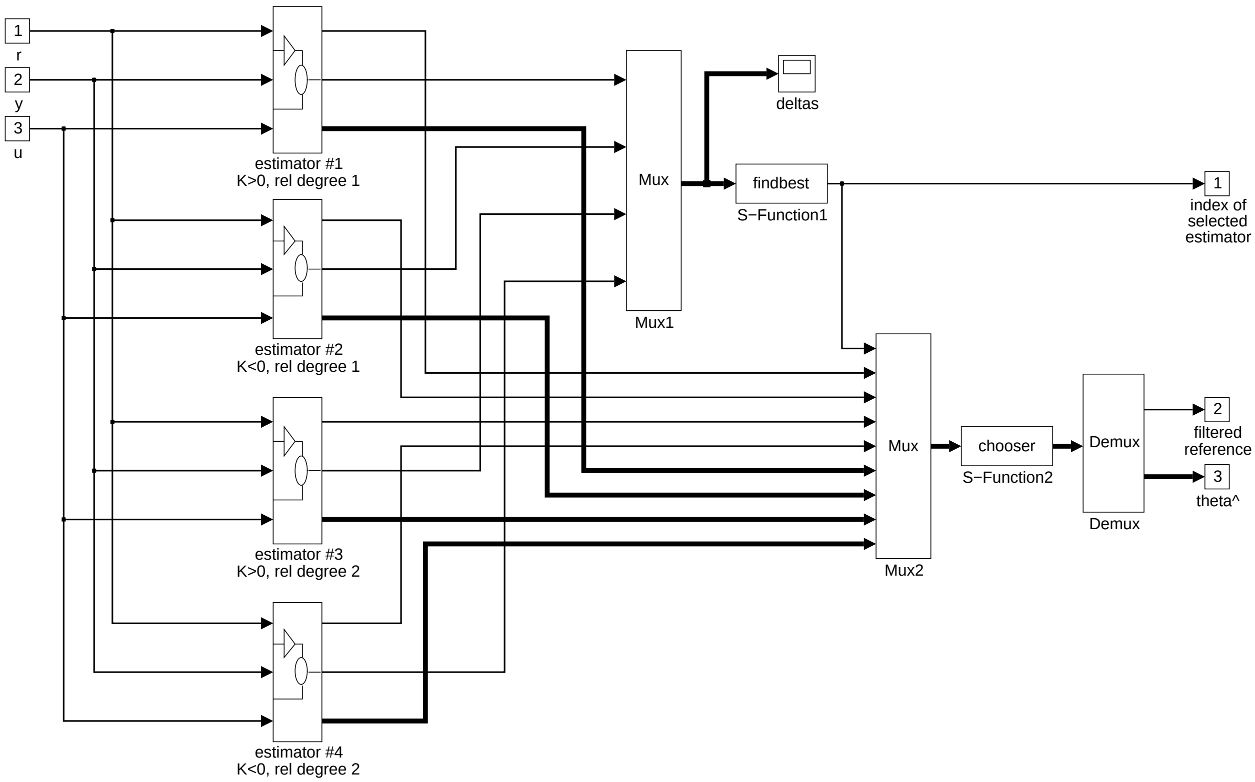
test function δ_i small $\Rightarrow$ ith estimator is doing well



- **Hysteresis switching adaptive control**

- Construct a supervisory control system which monitors the test functions associated with each estimator, and use the parameter estimate $\hat{\theta}$ produced by that estimator which is doing "best"

- Switch estimators when one is doing better than the currently selected estimator by an amount h, the *hysteresis constant*

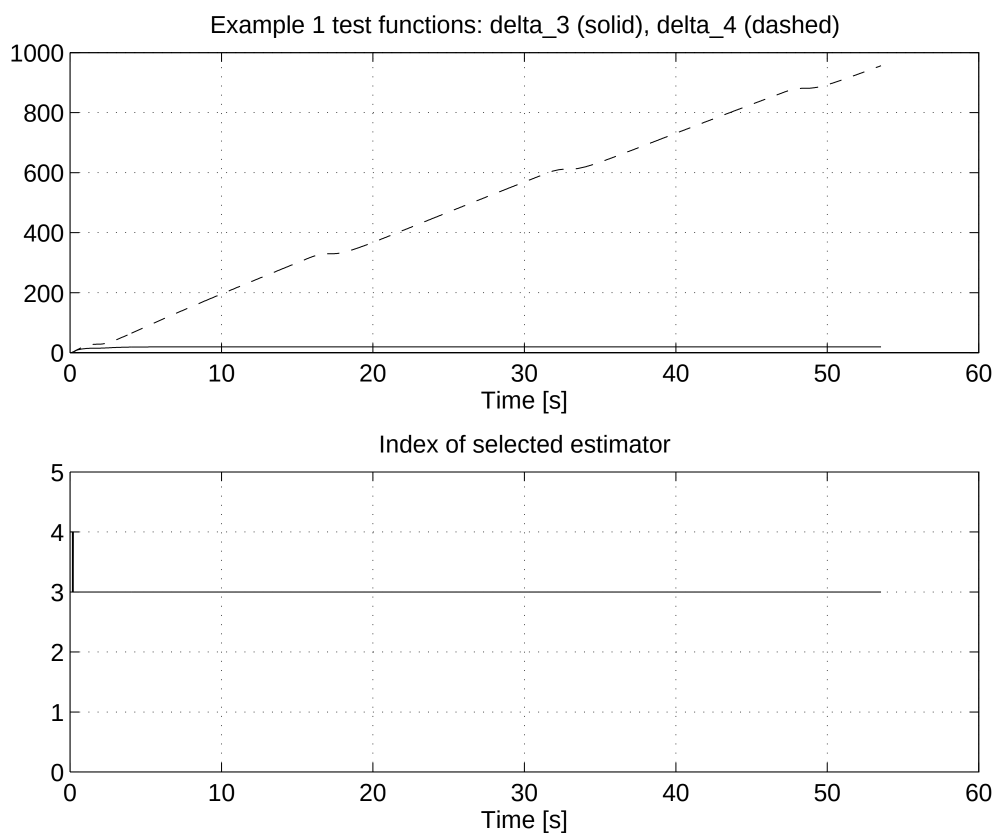


- Utilising hysteresis in the switching logic achieves two (related) goals:

  1. it prevents unbounded "chattering"

  2. since the test functions are well-behaved and at least one (corresponding to the "correct" estimator) is bounded, the Hysteresis Switching Lemma guarantees that all switching stops after a finite time T^*

3. Example and SIMULINK demonstration



4. Hysteresis switching adaptive control of MIMO plants

• How can the notions of "relative degree" and "high-frequency gain" be generalized to multi-input multi-output (MIMO) plants?

One way is via the *interactor matrix*, a lower triangular polynomial matrix $\xi(s)$ such that

$$\lim_{s\to\infty} \xi(s)P(s) = K, \qquad \det(K) \neq 0$$

Compare this with the SISO case:

$$\lim_{s\to\infty} s^d P(s) = k, \qquad k \neq 0$$

Example

$$P(s) = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{1}{s+2} \\ \dfrac{1}{s+4} & \dfrac{1}{s+3} \end{bmatrix} \qquad \text{plant transfer matrix}$$

$$\xi(s) = \begin{bmatrix} s+a & 0 \\ -(s+a)^2 & (s+a)^2 \end{bmatrix} \qquad \text{interactor matrix}$$

$$K = \lim_{s\to\infty} \xi(s)P(s) = \begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix} \qquad \text{high-frequency gain matrix}$$

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• There are serious difficulties in extending the hysteresis switching scheme to the multivariable case:

- the "relative degree" may now have contain *real-valued* parameters
- What sort of prior knowledge of K is required to ensure estimates of K remain nonsingular?

Interactor matrix

- **Problem:** The "relative degree" information in the interactor may have real-valued entries

$\Rightarrow$ "countability" problem arises

- **Solution:** Estimate the real-valued entries as part of the parameter matrix

- Write interactor matrix as

$$\xi(s) = \Psi(s) + \Delta(s)$$

where

  - $\Psi(s)$ is a strictly lower triangular polynomial matrix, with $\partial(\Psi) \leq d$:

$$\Psi(s) = \Psi_d s^d + \cdots \Psi_1 s + \Psi_0$$

  - $\Delta(s) = \mathrm{diag}[(s+a)^{n_1}, \ldots, (s+a)^{n_m}]$

- Assuming integers $n_1, \ldots, n_m$ are known, can estimate $\Psi_0, \ldots \Psi_d$ matrices. When $n_1, \ldots, n_m$ are unknown, need to run d^m estimators:

  - m integers $n_1, \ldots, n_m$
  - each n_i is in the range $1 \leq n_i \leq d$

High-frequency gain matrix

Assume K has an LU (lower-triangular upper-triangular) decomposition:

$$K = QR$$

where

- Q is lower triangular with 1's on the main diagonal
- R is upper triangular with diagonal entries r_{ii}

Then

$$\det(K) \neq 0 \qquad \Longleftrightarrow \qquad r_{ii} \neq 0$$

since

$$\det(K) = \det(Q)\det(R) = \det(R) = \prod_{i=1}^{m} r_{ii}$$

• Which which of the 2^m alternatives for the signs of r_{ii} is correct? Run 2^m estimators, each assuming a different permutation of signs and incorporate projection as before

• To ensure an LU decomposition of K exists, must have

$$K^{[1]}, K^{[2]}, \cdots, K^{[m-1]} \quad \text{nonsingular}$$

where $A^{[k]}$ is the leading principal submatrix of order k of A

• Consider all $m!$ permutations of input vector components:

- interactor matrix unaffected by input permutations
- columns of high-frequency gain matrix are permuted
- since K nonsingular, at least one input permutation is such that resulting K has an LU decomposition

• To ensure nonsingularity of estimated K, can run $2^m \times m!$ estimators in parallel

- Now write

$$L^{-1}\Delta[y] = \theta^T \psi$$

where

$$\theta^T = [\tilde{Q},\ R,\ -\bar{\Psi}_d,\ \cdots,\ -\bar{\Psi}_0,\ \tilde{W}_\nu,\ \cdots,\ \tilde{W}_0\ \tilde{S}_{\nu-1},\ \cdots,\ \tilde{S}_0]$$

- Number of parallel estimators = $d^m \times 2^m \times m!$

- Each estimator corresponds to a particular permutation of
  - polynomial degrees of diagonal entries of interactor matrix
  - signs for the diagonal entries of R
  - entries of plant input vector

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