Example: Satellite Tracking

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Tracking geostationary satellites

- geostationary communications satellites:
  - positioned ≈ 35,800 km above Earth's surface
  - spaced around equatorial plane
- in theory:
  - orbit Earth with period of rotation = one sidereal day = 23 hours, 56 minutes

  ⇒ synchronous with Earth's rotational period

  ⇒ apparently stationary to observer on Earth's surface
- in practice:
  - orbit perturbed by:
    - Earth's non-uniform gravitational field
    - gravitational attraction of Sun and Moon
    - solar radiation pressure

  ⇒ satellites "wobble" with period of one sidereal day

  ⇒ need to track satellite for maximum received signal strength

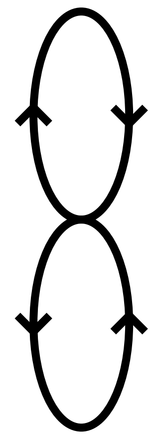


Figure-of-8 traced out on Earth's surface

Open-loop tracking

1. use physical modeling of satellite motion (with perturbations) to predict satellite motion into future
2. periodically adjust antenna pointing angles

- routinely used in practice, but:
  - requires high absolute accuracy in the antenna azimuth–elevation encoders
  - requires regular maintenance to put in new model parameters
    - in practice: once a week, satellite operators (eg. Intelsat) fax hourly position updates for specified satellite and ground station latitude–longitude–height
  - unable to compensate for wind and thermal effects on antenna

Pointing data

sydney 1a EARTH STATION DESIGNATOR
-33.731389 EARTH STATION NORTH LATITUDE (DEGREES)
151.229722 EARTH STATION EAST LONGITUDE (DEGREES)
.076000 EARTH STATION HEIGHT ABOVE IAU-1977 ELLIPSOID (KM)

60.00 TIME INTERVAL BETWEEN AZ/EL POINTS (MINUTES)
7.0 DURATION OF POINTING INFORMATION GENERATION (DAYS)

180.00 CENTER OF BOX .02 LONGITUDE BOX .02 LATITUDE BOX

EPHEMERIS VALUES

LM0	=	179.99070	179.9907000
LM1	=	-.10700E-01	-.0107000
LM2	=	.79700E-03	.0007970
LONC	=	-.64000E-02	-.0064000
LONC1	=	-.60000E-03	-.0006000
LONS	=	.53100E-01	.0531000
LONS1	=	-.20000E-03	-.0002000
LATC	=	-.33600E-01	-.0336000
LATC1	=	-.70000E-03	-.0007000
LATS	=	.70700E-01	.0707000
LATS1	=	-.39000E-02	-.0039000

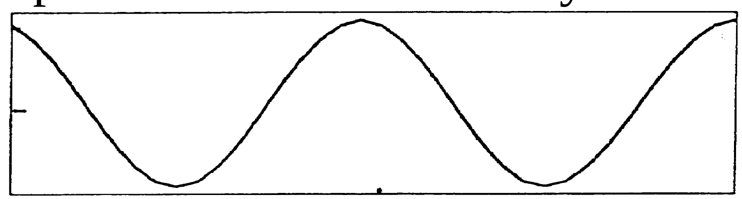
EPOC DATE 3/12/88 OF EPHEMERIS (DAY/MONTH/YEAR)
EPOC TIME 0: 0: 0 OF EPHEMERIS (HOUR:MINUTES:SECONDS)

PREDICTED SATELLITE POSITION AT +170 HOURS = 179.977 -.005
CALCULATED SATELLITE POSITION AT +170 HOURS = 179.977 -.005

ELAPSED HOURS	YR:MO:DA:HR:MN:SC	AZ(DEG)	EL(DEG)	SAT LONG (DEG)	SAT LAT (DEG)	
.0	88:12: 3: 0: 0: 0	44.715	40.241	179.9843	-.0336	
1.0	88:12: 3: 0:59:60	44.714	40.215	179.9978	-.0142	**
2.0	88:12: 3: 2: 0: 0	44.712	40.188	180.0108	.0061	**
3.0	88:12: 3: 3: 0: 0	44.709	40.163	180.0224	.0260	
4.0	88:12: 3: 3:59:60	44.705	40.142	180.0317	.0440	
5.0	88:12: 3: 5: 0: 0	44.700	40.125	180.0381	.0590	
6.0	88:12: 3: 6: 0: 0	44.694	40.114	180.0411	.0699	
7.0	88:12: 3: 6:59:60	44.688	40.110	180.0405	.0760	
8.0	88:12: 3: 8: 0: 0	44.682	40.113	180.0363	.0770	
9.0	88:12: 3: 9: 0: 0	44.677	40.122	180.0288	.0727	
10.0	88:12: 3: 9:59:60	44.672	40.138	180.0184	.0636	
11.0	88:12: 3:11: 0: 0	44.669	40.158	180.0058	.0501	
12.0	88:12: 3:12: 0: 0	44.667	40.183	179.9918	.0334	

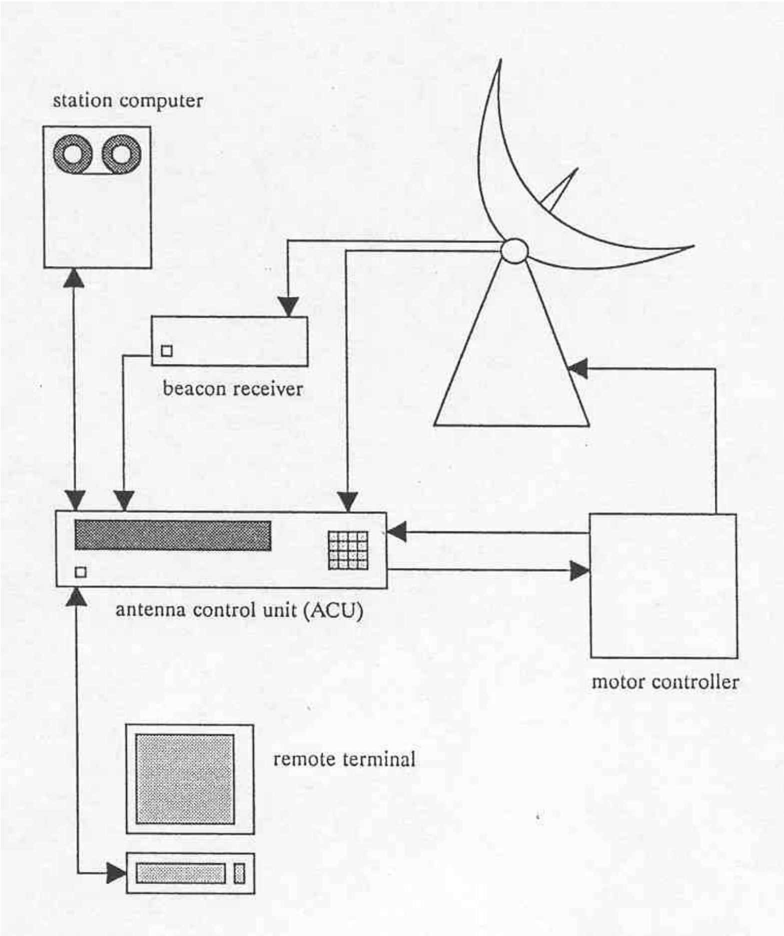
Plot azimuth or elevation pointing data and…

it's *very* close to sinusoidal with period of one sidereal day



time

Closed-loop tracking



❖ *Idea*

move antenna periodically to find direction of maximum beacon signal strength

❖ *Problem*

Measured beacon signal is unreliable:

- noise on the received signal
- variations in signal intensity transmitted from satellite
  - not *meant* to happen but…
- effect of wind gusts on the structure

A linear model

Pointing angle data suggests models for satellite *azimuth* and *elevation* variation with time *t*:

$$\alpha(\boldsymbol{t}) = \alpha_0 + \alpha_1 \boldsymbol{t} + \alpha_2 \boldsymbol{t}^2 + (\alpha_3 + \alpha_4 \boldsymbol{t})\cos \omega \boldsymbol{t} + (\alpha_5 + \alpha_6 \boldsymbol{t})\sin \omega \boldsymbol{t} + \alpha_7 \cos 2\omega \boldsymbol{t} + \alpha_8 \sin 2\omega \boldsymbol{t} + \cdots$$

$$\varepsilon(\boldsymbol{t}) = \varepsilon_0 + \varepsilon_1 \boldsymbol{t} + \varepsilon_2 \boldsymbol{t}^2 + (\varepsilon_3 + \alpha_4 \boldsymbol{t})\cos \omega \boldsymbol{t} + (\varepsilon_5 + \varepsilon_6 \boldsymbol{t})\sin \omega \boldsymbol{t} + \varepsilon_7 \cos 2\omega \boldsymbol{t} + \varepsilon_8 \sin 2\omega \boldsymbol{t} + \cdots$$

- fundamental period of one sidereal day $\Rightarrow$ ω known
- coefficients α_i and ε_i unknown but approx. constant

A simplified linear model

Several terms present, but each axis dominated by constant + fundamental:

$$\alpha(\boldsymbol{t}) \approx \alpha_0 + \alpha_3 \cos \omega \boldsymbol{t} + \alpha_5 \sin \omega \boldsymbol{t}$$

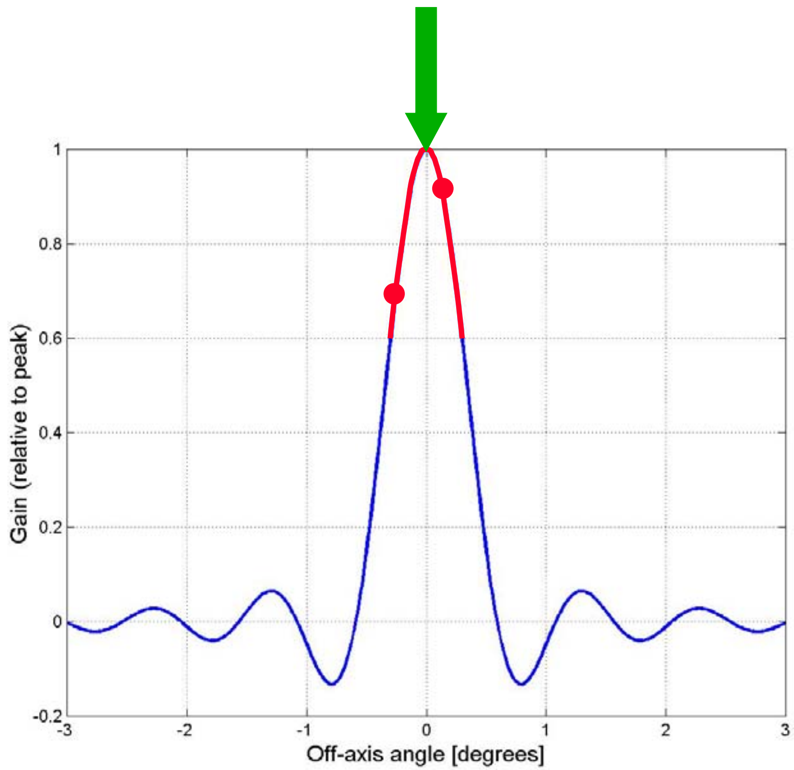
$$= \begin{bmatrix} 1 & \cos \omega \boldsymbol{t} & \sin \omega \boldsymbol{t} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_3 \\ \alpha_5 \end{bmatrix}$$

Observation: expression has the form of output equation for state-space model:

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}(\boldsymbol{t})\boldsymbol{x}$$

Idea: use KF to estimate state of system whose states are α_i

From beacon power to azimuth–elevation



❖ *Problem*

To apply Kalman filter to estimate α_i we need measurements of actual satellite azimuth

- but we only have beacon strength measurements

❖ *Solution*

Use known shape of beam pattern to infer satellite azimuth from just two measurements of beacon power

- beamwidth depends only on frequency of comms link, and diameter of antenna

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System model

State variables assumed constant:

$$\boldsymbol{x}_1 = \alpha_0, \quad \boldsymbol{x}_2 = \alpha_3, \quad \boldsymbol{x}_3 = \alpha_5$$

$\Rightarrow$ states "propagate" according to:

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \end{bmatrix} = 0$$

States are measured via:

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}(\boldsymbol{t})\boldsymbol{x}(\boldsymbol{t}), \qquad \boldsymbol{C}(\boldsymbol{t}) = [1 \quad \cos\omega\boldsymbol{t} \quad \sin\omega\boldsymbol{t}]$$

Modified system model

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad + w(t)$$

process noise, models:

a) known undermodelling

b) slow drift of "constant" parameters

$$E\{w(t)w^T(t)\} = Q$$

$$y(t) = C(t)x(t) + v(t)$$

measurement noise

$$E\{v^2(t)\} = R$$

Need a model like this for both azimuth *and* elevation axes

Application of Kalman filter

Given:

- modified system model
- process noise covariance Q and measurement noise variance R
- noisy measurements of satellite azimuth, $y(t)$

Compute:

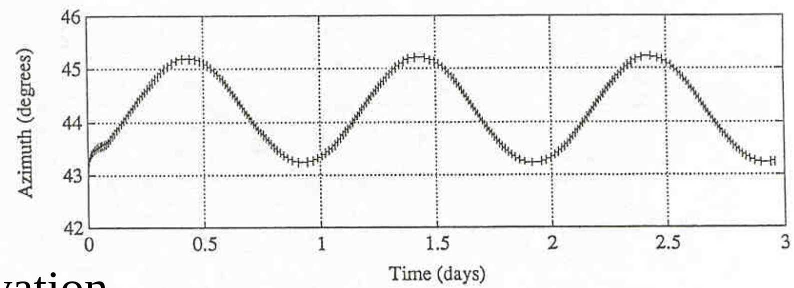
- optimal estimates of unknown state variables $\boldsymbol{x}_1 = \alpha_0, \boldsymbol{x}_2 = \alpha_3, \boldsymbol{x}_3 = \alpha_5$ using Kalman filter
- predicted satellite azimuth $\hat{\alpha}(\boldsymbol{t}) = \alpha_0 + \alpha_3 \cos \omega \boldsymbol{t} + \alpha_5 \sin \omega \boldsymbol{t}$

Getting it to work in practice

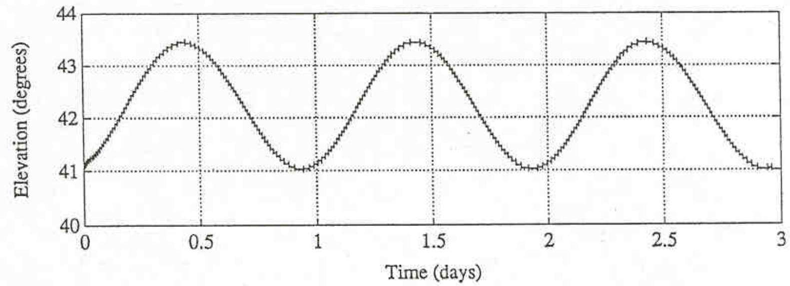
- *How do you know what the process noise covariance matrix* ***Q*** *is?*
  - We don't. It's estimated based on prior knowledge of likely undermodeling and parameter drift. Too big and KF ignores model; too small and KF ignores measurements. In the end, ***Q*** is a tuning knob.
- *How do you know what the measurement noise variance* ***R*** *is?*
  - It's estimated based on sample variance of measured beacon power.
- *How do you calculate covariance of initial state estimation error* ***P(0)****?*
  - Ummm, we make a guess, assigning larger values to values corresponding to parameters we expect are more important in model. Choosing ***P(0)*** too small can cause KF to take a long while to start noticing measurements.
- *What do you actually do with estimates of satellite azimuth and elevation?*
  - Predict ahead a little, then wait for satellite to catch up, then move on. Repeat.
- *What do you mean by "a little"?*
  - That depends on confidence in estimates, measured by size of state estimation error covariance matrix $\boldsymbol{P} = \boldsymbol{E}\left\{(\boldsymbol{x} - \hat{\boldsymbol{x}})(\boldsymbol{x} - \hat{\boldsymbol{x}})^{\boldsymbol{T}}\right\}$

Does it work? Yes!

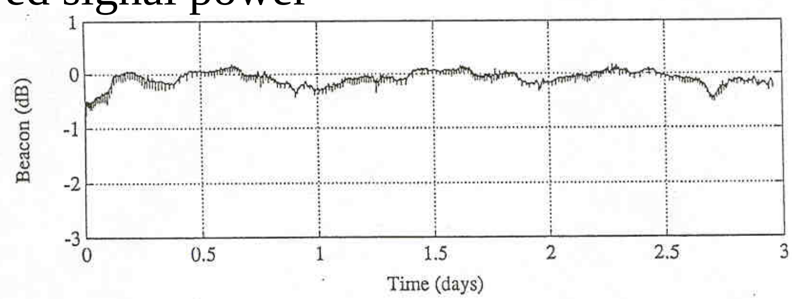
azimuth



elevation



received signal power

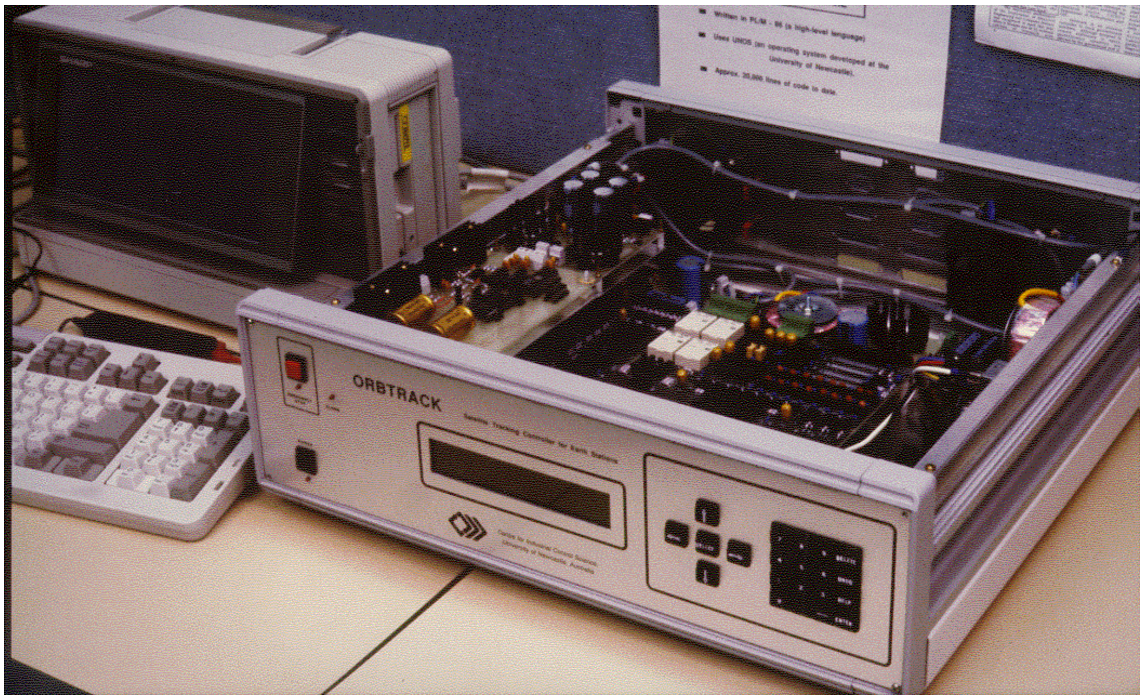


- ❖ measured data from 3 days of tracking at OTC (now Telstra) ground station at Oxford Falls, Sydney
- ❖ C-band (4-GHz) and 8m diameter antenna

⇒ half-power beamwidth of 0.66 degrees

- ▪ less than 0.5dB variation in received signal, despite azimuth/elevation variations of ≈ 2 degrees

ORBTRACK



- commercial system based on these principles designed and built at the University of Newcastle, Australia
- system marketed under the trade name ORBTRACK®
- used in many real-world applications in Australia, Indonesia, Antarctica and <prefix>-istan